Preferential attachment e clustering

Estendere l'algoritmo di Barabási-Albert

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Clustering

- “Transitività” nella rete.
- Due definizioni di clustering:

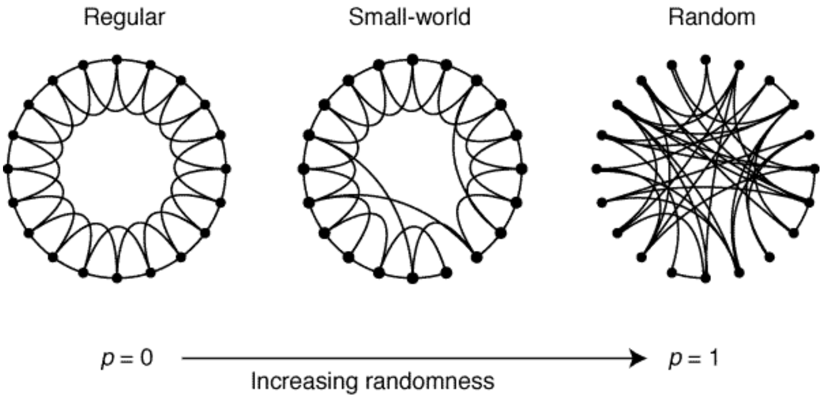
$$C_0(G) = \frac{3 \cdot \text{numbero di triangoli in } G}{\text{numero di triple connesse in } G}$$

$$c_1(v) = \frac{\text{numbero di triangoli connessi a } v}{\text{numero di triple centrate su } v}$$

$$C_1(G) = \frac{1}{n} \sum_{i \in V} c_1(i).$$

Small World

- Watts-Strogatz, *Nature*, 1998.
- Reti con alto clustering e basso diametro.
- Modello: lattice, salti a lunga distanza.



Scale Free

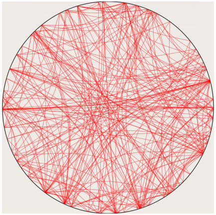
- Barabási-Albert, *Science*, 1999.
- Distribuzione power-law dei gradi dei nodi.
- Rete in crescita: ogni nuovo nodo crea una connessione ad un vecchio nodo.
- Preferential attachment:

$$\Pi\left(k_j\right) \sim k_j.$$

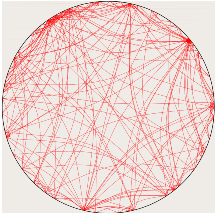
Algoritmo (1)

- Nodi disposti casualmente su un anello.
- Preferential attachment:

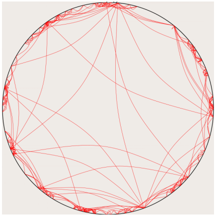
$$\Pi\left(k_j, d_{ij}\right) \sim \frac{\left(k_j\right)^{\alpha}}{\left(d_{ij}\right)^{\sigma}}.$$



$\alpha = 1,\ \sigma = 0$ (BA)



$\alpha = 1,\ \sigma = 1$

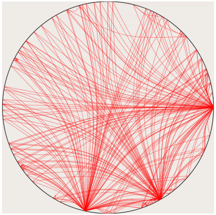


$\alpha = 1,\ \sigma = 3$

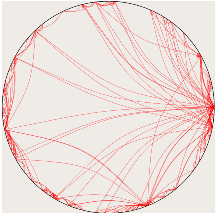
Algoritmo (2)

- Definiamo $\gamma = \frac{\sigma}{\alpha}$: $\Pi(k_j, d_{ij}) \sim \left(\frac{k_j}{(d_{ij})^\gamma}\right)^\alpha$.
- Limite per $\alpha \to \infty$:

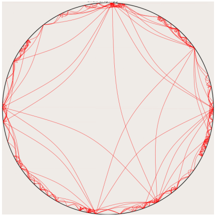
$$\arg\max_{j \in V} \frac{k_j}{(d_{ij})^\gamma}.$$



$\gamma = 0.5$

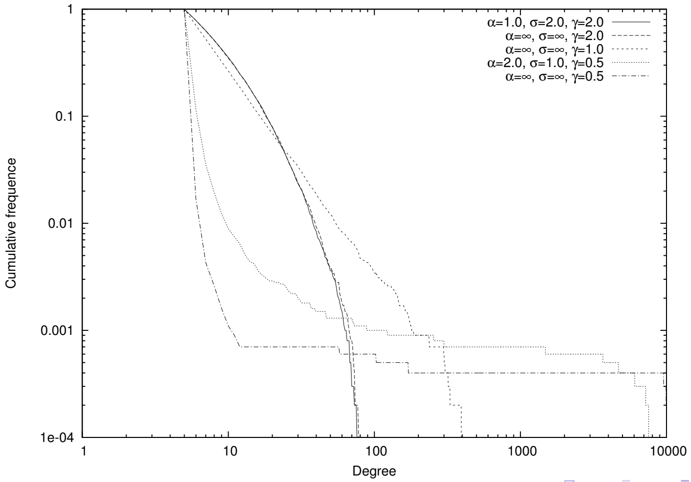


$\gamma = 1$

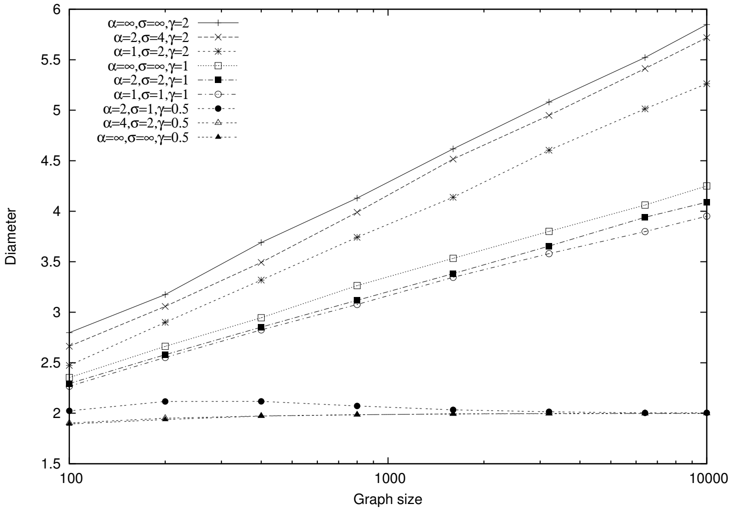


$\gamma = 2$

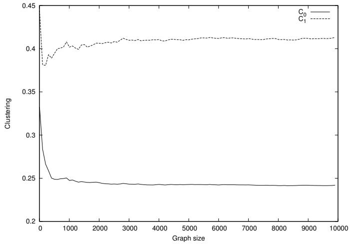
Grado


α=1.0, σ=2.0, γ=2.0
α=∞, σ=∞, γ=2.0
α=∞, σ=∞, γ=1.0
α=2.0, σ=1.0, γ=0.5
α=∞, σ=∞, γ=0.5
Cumulative frequence
Degree
1
0.1
0.01
0.001
1e-04
1
10
100
1000
10000

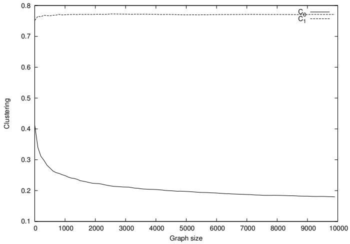
Diametro


α=∞,σ=∞,γ=2
α=2,σ=4,γ=2
α=1,σ=2,γ=2
α=∞,σ=∞,γ=1
α=2,σ=2,γ=1
α=1,σ=1,γ=1
α=2,σ=1,γ=0.5
α=4,σ=2,γ=0.5
α=∞,σ=∞,γ=0.5
Diameter
Graph size
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
100
1000
10000

Evoluzione e clustering

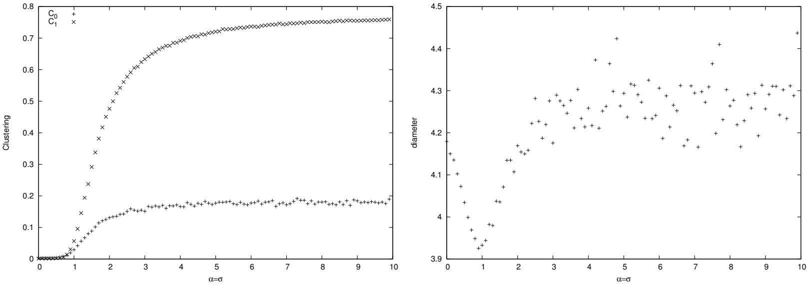


$m = 5,\ \alpha = 1,\ \sigma = 2$



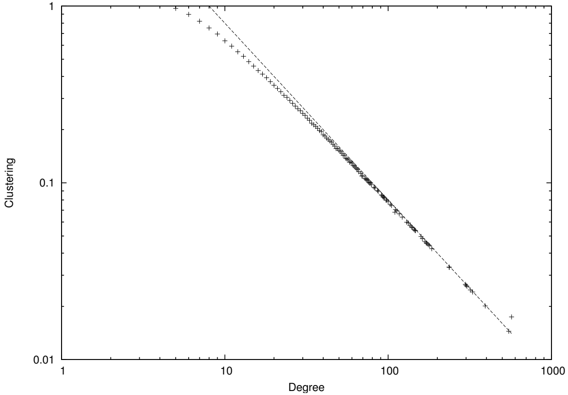
$m = 5,\ \alpha = \sigma = \infty,\ \gamma = 1$

$\gamma = 1$


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Clustering
C_0
C_1
0 1 2 3 4 5 6 7 8 9 10
α=σ
4.5
4.4
4.3
4.2
4.1
4
3.9
diameter
0 1 2 3 4 5 6 7 8 9 10
α=σ

Gerarchia

- Reti gerarchiche: $c_1(v) \sim k_v^{-1}$ (Ravasz-Barabási, 2003).



Riassunto

- Famiglia di modelli per modellare reti complesse.
- Estensione del modello di Barabási-Albert.
- Caratteristiche ricorrenti: small world, scale-free, gerarchia.

- Sviluppi futuri
  - Studio analitico
  - Altre topologie