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**Reliability and QoS Analysis of the Italian GARR network**  
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# Reliability and QoS Analysis of the Italian GARR network

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## Abstract

The present technical report is aimed at discussing the modeling, analysis and computation of the reliability and quality of service (QoS) of complex networks, with particular emphasis on the Italian GARR network. The present report is primarily based on the methodologies described in previous work, where the algorithm for deriving the reliability of a binary probabilistic network, via the use of BDD is described. Moreover, the present report introduces new capabilities and analysis techniques, by considering weighted networks. Weight are parameters that characterize the service that the network is able to provide, so that the analysis of weighted can give insight on the service availability provided by any system representable in the form of a network.

## 1 Introduction

The present technical report describes an expanded set of tools for the analysis of complex networks. The reports starts form the Network Reliability Analyser (NRA) tool described in [3] and moves forward in the direction of enriching the the system description and the measures that can be obtained, by including in the definition and characterization of the system some parameters or weights related to the services that the network can provide, and by computing the availability of the service or the Quality of Service (QoS) associated to the network.

To this end, the specification of the network must be augmented with an index or a weight that indicates a service feature of the network. We show that the weights can have many different physical meanings, and

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correspondingly require a different treatment. In particular, the weights can indicate a characteristics related to a cost or a distance or a resistance (i.e. a property that is additive along the paths) or a capacity (i.e. a property that is additive along the cuts). For this step, we define formally the concept of a "*Probabilistic Weighted Network*" and we discuss the related QoS measures and how to compute them. To deal with the analysis of weighted networks we extend the data structure of the BDD's into a new and more powerful data structure called Algebraic Decision Diagrams ADD.

The analysis is specifically directed toward the evaluation of critical systems in the form of networks and in the interrelation among networks. To show the potentialities of the developed methodology and of the implemented tool we carry our analyses and computations on a publicly available case study. In particular, we concentrate our attention to the Italian GARR Network. The GARR Network (whose acronym means "Gestione Ampliamento Rete Ricerca" - "Research Network Widening Management") is composed by all subjects representing the Italian Academic and Scientific Research Community, and manages their internet services. The layout of the GARR network and its main bandwidth characteristics are available from: <http://www.garr.it/>.

The aim of the present report is to show the potentialities of the set of analysis techniques developed at our department for the study of the reliability and Quality of Service of complex networks, based on the illustration of a case study taken from available net structures downloaded from public web sites.

## 2 The Italian GARR network

The Italian Academic and Research Network GARR (<http://www.garr.it/index.php>) is based on scientific and academic collaboration projects between Italian Universities and public Research Organisations. The GARR network service is mainly provided for the GARR community.

Figure 1 shows the layout of the major connections of the GARR network. The network in the picture is composed of 42 nodes and 52 links.

The following measures can be computed on the network using the tool NRA:

1. *(s-t) Network reliability* - Given a source node  $s$  and a sink node  $t$  the  $(s-t)$  Network reliability is the probability that the source and the sink are connected by at least one path of working edges.

Table 1: Reliability computations on the graph of Figure 1

Source Node	Terminal Node	Minpath	Mincut	No. BDD Nodes	Reliability		
					Single Edge		
					0.9	0.95	0.99
TO	CT	196	481	1003	0.977428	0.994713	0.999797
TS1	NA	168	385	604	0.975771	0.994486	0.999795

2. *List of minpaths* - Given a network and a source node  $s$  and a sink node  $t$ , a path  $H$  between  $s$  and  $t$  is a subset of edges that guarantees the source  $s$  and sink  $t$  to be connected if all the edges of the subset are functioning. A path  $H$  is a minpath if a subset of elements in  $H$  does not exist that is also a path. The NRA tool provides the list of all the minpaths ordered according to their order (number of edges forming the path).
3. *List of mincuts* - Given a network and a source node  $s$  and a sink node  $t$ , a cut  $K$  between  $s$  and  $t$  is a subset of edges whose failure disconnects the source  $s$  and sink  $t$ . A cut  $K$  is a mincut if a subset of elements in  $K$  does not exist that is also a cut. The NRA tool provides the list of all the mincuts ordered according to their order (number of edges forming the cut).

The analysis technique used by the tool NRA to obtain the above measures is based on the representation of the connectivity function of the network by means of a binary tree called Binary Decision Diagram (BDD) as illustrated in [2, 3].

## 2.1 Reliability analysis of the GARR network

We study the properties of the GARR network in two cases assuming two different couples of source and sink nodes: *i)* - in the first case we assume the node  $TO$  as a source and the node  $CT$  as sink; *ii)* - in the second case we assume the node  $TS1$  as a source and the node  $NA$  as sink (see Figure 1). In the lack of available data about the reliability of the single elements of the GARR network, and with the goal of performing quantitative computations, we have assumed, arbitrarily, a uniform value  $p$  for the arcs of being up ( $1 - p$  down). Usually we take  $p = 0.9$ . A summary of the results is presented in Table 1.

The first two columns of Table 1 indicate the chosen source and sink nodes: the first row reports the

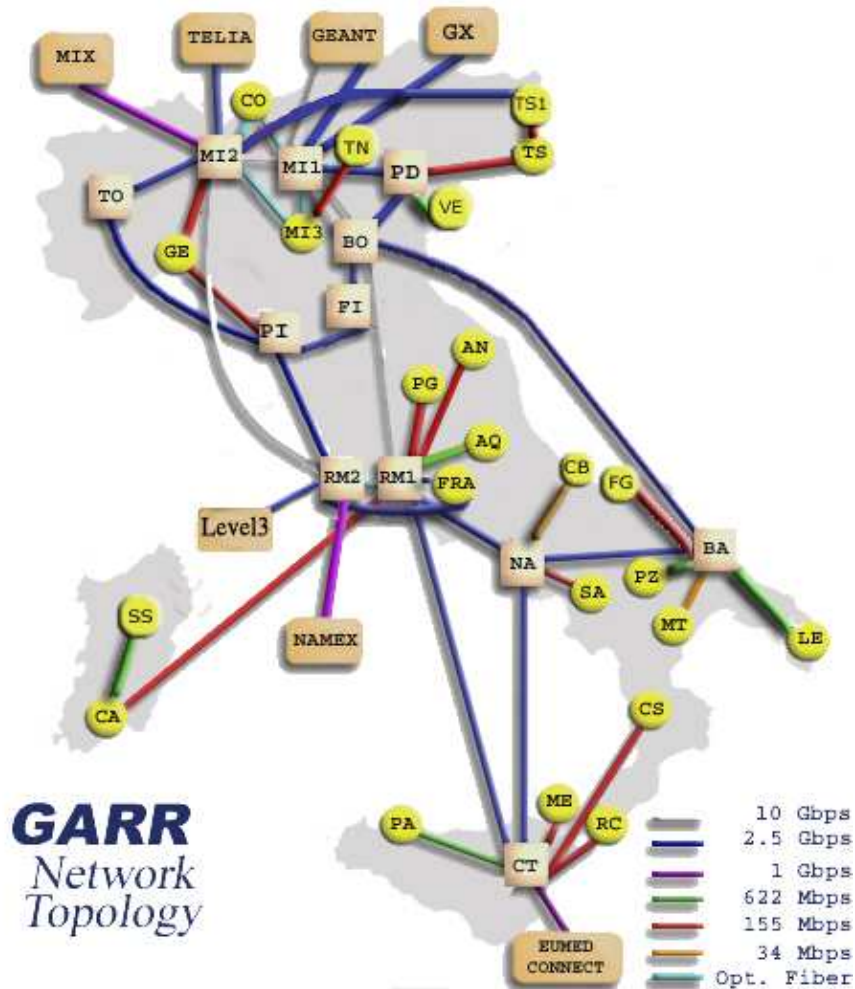


Figure 1: GARR network

first examined case, the second row the second case. The subsequent columns report the results obtained by running the NRA tool. In detail: the number of *minpaths*, the number of *mincuts* and the number of BDD nodes generated by using the generation algorithm based on a depth-first search on the graph [2, 3]. The final three columns report the (s-t) reliability value assuming that only the arcs can fail, all with identical probability  $p$  ( $p = 0.9, 0.95, 0.99$ , respectively).

The connectivity properties depend not only on the number of *minpaths* (*mincuts*) but also on their order, that is the number of edges that form the *minpath* (*mincut*). To show the order, we have derived the histograms of the distribution of the length of the *minpaths* and *mincuts* for the different s-t cases reported

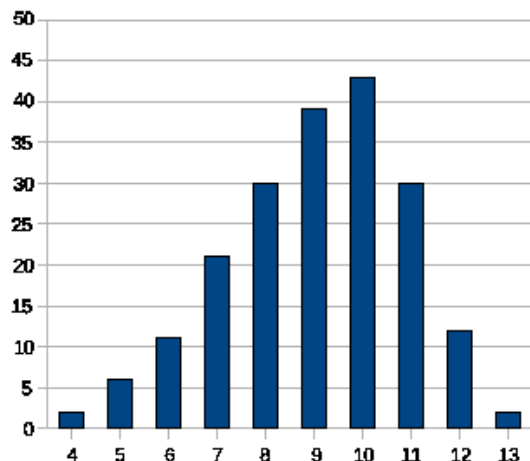


Figure 2: Histogram of the minpath length for the case  $s=TO$   $t=CT$

in Table 1. For the case  $s=TO$   $t=CT$ , Figure 2 reports the histogram of the length of the *minpaths* and Figure 3 the histogram of the order of the mincuts. It is evident that the minpaths of lower order are the most favorable to connect source and sink, while the mincuts of lower order are the most critical for interrupting the connection between source and sink. In any case, the tool NRA provides the complete list of edges that form each minpath and its probability of being up, and the complete list of edges that form each mincut and its probability of being down.

Similar results are reported in the next two figures for the case  $s=TS1$   $t=NA$ . Figure 4 reports the histogram of the length of the minpaths, while Figure 5 reports the histogram of the order of the mincuts.

### 3 Weighted Networks

Traditional studies on network reliability, as those exemplified in Section 2.1, assume that nodes and links are binary entities (either up or down) and that the node-to-node reliability is computed as the probability that there exists at least one path of working links connecting the two nodes. In many cases a richer representation is useful or even needed, when some property associated to the edges (capacity, cost, length) influences the performance of the system. For example, the amount of traffic characterizing the connections in communication or transport systems, or the distance between nodes in a highway network, or the resistance of

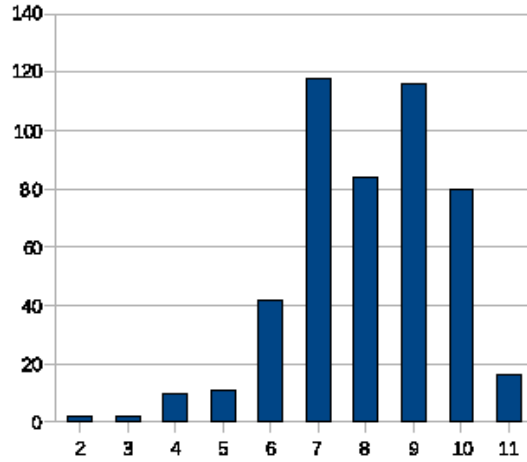


Figure 3: Histogram of the mincut order for the case  $s=TO$   $t=CT$

the branches in a electrical grid, are fundamental parameters for a full description of these systems. Motivated by these observations, we consider weighted probabilistic networks whose edges have been assigned a weight (indicated as a label associated to the edge). We distinguish between two types of interpretation of weights which can be used for characterizing the properties of a network.

- i)* Weights are interpreted as costs, or lengths or resistances associated to the edges so that the aim of the analysis is to evaluate the probability that the connection can be established below a given cost (or distance or resistance).
- ii)* Weights are interpreted as capacities or bandwidths, representing the maximum flow that the edge can support when up. The aim of the analysis is addressed to the ability of the network to transport a given amount of flow.

Many analysis techniques have been explored in the literature for computing the reliability in binary probabilistic networks. In the case of dependable weighted networks, a more informative measure is the Quality of Service (QoS) defined, in the two cases above, as:

- i)* the probability that the source node and the sink node can be reached with a cost (or a distance) not greater than a given threshold;

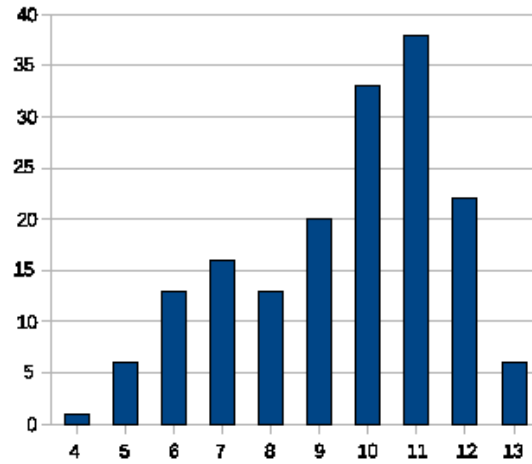


Figure 4: Histogram of the minpath length for the case  $s=TS1$   $t=NA$

*ii)* the probability that the network is able to transport a flow from the source to the sink not less than a given demand.

As in binary probabilistic networks the most efficient representation of the network connectivity function seem to be based by means of Binary Decision Diagrams (BDD) [4, 2], the analysis of weighted probabilistic networks can be based on a data structure, derived from BDD, and called Algebraic Decision Diagram (ADD). An ADD is a binary tree whose terminal leaves can assume any positive value between 0 and the maximum flow (or distance) in the network.

A path connecting the root of the ADD with a terminal leaf with label, say  $\alpha$ , indicates that a flow (or distance) equal to  $\alpha$  is transported along this path. The associated probability can be easily computed from the ADD.

## 4 Probabilistic Weighted Networks

A weighted stochastic network is a tuple  $N = (G, P, W)$  where  $G = (N, E)$  is a Graph (with  $V$  vertices and  $E$  edges),  $P$  is the probability function that assigns to each edge  $e(u, v)$  ( $u, v \in V$ ) a probability  $p(u, v)$  of being up (and  $1 - p(u, v)$  of being down) and  $W$  is a weight function that assigns to each edge  $e(u, v)$  a finite real weight  $w(u, v)$ .  $w(u, v)$  has the meaning of a reward assigned to the arc that qualifies and quantifies



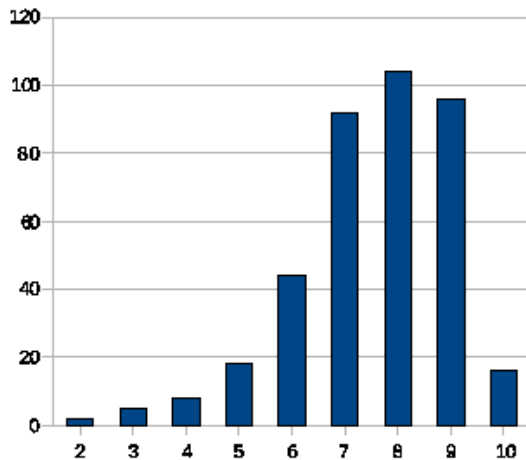


Figure 5: Histogram of the mincut order for the case  $s=TS1$   $t=NA$

the service carried by the arc, or its cost, or some other attribute of the arc. Given a source node  $s$  and a termination node  $t$ , we can evaluate the Quality of Service (QoS) between  $(s-t)$  as a reward measure that is a function of the structure of the graph, of the probability function  $P$  and of the weight function  $W$ . Let us denote this QoS measure as  $\Psi_{s,t}$ . We consider two cases.

#### 4.1 The Weight is a Cost

By cost we identify, generically, a parameter that is additive along the arcs of the network. From this point of view, a cost can be: a real monetary cost, the length of the arc, the resistance of the arc. The weight  $w(u, v)$  of the arc  $e(u, v)$  is a cost function related to the input and output node  $(u, v)$  of the arc. A simple physical idea is the length of the arc, i.e. the distance between nodes  $u$  and  $v$ . The reward function  $\Psi_{s,t}$  is the minimum cost between  $s$  and  $t$  computed as the sum of the costs of the arcs of a path connecting  $s$  and  $t$ . In a stochastic network the problem can be reformulated as a QoS problem in the following terms.

*Problem 1 - Given a stochastic weighted network  $N=(G,P,W)$ , compute the probability that the cost  $\Psi_{s,t}$  between  $s$  and  $t$  is below a threshold  $\psi_{max}$ .*

To solve *Problem 1* we need some definitions.

*Definition 1 - Given a stochastic network  $N = (G, P, W)$  and a source node  $s$  and a sink node  $t$ , the cost of a path  $H(s, t)$  is the sum of the costs  $w(u, v)$  of the edges forming the path.*

*Corollary 1 min-distance* - If a network  $N = (G, P, W)$  has  $n$  minpaths  $H_1, H_2, \dots, H_n$  the minimum cost between  $s$  and  $t$  is equal to the minimal cost of all its minpaths.

The min-cost corollary says that the minimal cost between any two nodes cannot be less than the minimal cost of all its paths.

## 4.2 Flow Networks

The weight  $w(u, v)$  of the arc  $e(u, v)$  is the nominal capacity or the bandwidth that the arc is able to carry. Networks with this attribute are usually called flow networks [5, 6] and the function  $\Psi_{s,t}$  is the maximum flow that can be transmitted from  $s$  to  $t$ . The maximum flow problem has received a great attention even in the recent literature. In stochastic networks the problem can be reformulated as a QoS problem in the following terms.

*Problem 2* - Given a stochastic flow network  $N=(G,P,W)$ , compute the probability that the flow  $\Psi_{s,t}$  guaranteed between  $s$  and  $t$  exceeds a minimum threshold  $\psi_{min}$ .

*Definition 2* - Given stochastic flow network  $N = (G, P, W)$ , and a source node  $s$  and a sink node  $t$ , the capacity of a cut  $K(s, t)$  is the sum of the capacities  $c(u, v)$  of all the edges forming the cut.

*Corollary 2 Max-flow Min-cut*- If a network  $N = (G, P, W)$  has  $m$  mincuts  $K_1, K_2, \dots, K_m$  the maximum flow between  $s$  and  $t$  is equal to the minimal capacity of all its mincuts.

The max-flow min-cut theorem says that the value of the maximum flow is equal to the minimal capacity carried by a mincut. In other words the theorem says that mincut that constitutes the bottleneck of a network determines its maximum flow. The quantity of flow between any two nodes cannot be greater than the weakest set of links somewhere between the two nodes.

## 4.3 Algebraic Decision Diagrams - ADD

Algebraic Decision Diagrams (ADD) (also called Multi Terminal BDD - MTBDD) are an extension of BDDs [1] which allow one to represent Boolean functions that can take any value, not just 0 or 1. ADDs represent a real function of  $n$  Boolean variables as a directed acyclic graph. In other words, BDDs can have only two

terminals  $(0, 1)$ , while ADDs can have more than two terminals; the terminal leaves of the ADD represent the value taken by the Boolean function along the path from the root to the terminal leaf.

Like BDD, ADD provide a compact representation of a Boolean expression by means of the Shannon's decomposition principle. Each node of the ADD represents a variable and has two successors: the left branch (in solid line) represents the value of the variable 1 and the right branch (in dotted line) represents the value of the variable 0. At each step we compute the QoS function so that at the end of the decomposition the final values of the  $\Psi_{s,t}$  function are obtained. The terminal leaves of the ADD are then all the possible values assumed by the  $\Psi_{s,t}$  function for any combination of 1 and 0 of the  $n$  Boolean variables on which the function is defined.

In the present case, we assume as the  $n$  Boolean variables the arcs of the graph and the  $\Psi_{s,t}$  function is defined according to *Corollary 1* or *Corollary 2* depending on the definition of the weights.

- If the weights are cost functions the terminal leaves of the ADD provide all the possible values of the costs computed along all the minpaths that are lower than the maximum threshold  $\psi_{max}$ . The value reported in any terminal leaf is the value of the  $\Psi_{s,t}$  function computed along any path connecting the root of the ADD with the terminal leaf.
- If the weights are capacity functions the terminal leaves of the ADD provide all the possible values of the flow computed along all the mincuts that are greater than the minimum threshold  $\psi_{min}$ . The value reported in any terminal leaf is the value of the  $\Psi_{s,t}$  function computed along any path connecting the root of the ADD with the terminal leaf.
- In both cases the single terminal leaf labeled with 0 is reached from the ADD root in all the cases in which the graph is disconnected or the  $\Psi_{s,t}$  does not respect the constraints.

The ADD is constructed by imposing an ordering of the variables. Similarly to BDDs, the size of the ADDs is extremely sensitive to the ordering of its variables. An ordered ADD can be reduced by successive applications of two rules. The *deletion rule* eliminates a node with both of its outgoing edges leading to the same successor. The *merging rule* eliminates one of two nodes with the same label as well as the same pair of successors. A reduced ADD with a fixed variable ordering is a canonical representation of a function. Thus, two identical functions will always have identical ADD representations under the same variable order. The

canonicity of ADDs gives rise to the existence of efficient algorithms for their manipulation.

In a probabilistic network  $N = (G, P, W)$ , each arc  $x_i$  is a Boolean variable with probability  $p_i$  of being up ( $1 - p_i$  down). At any node  $N$  of the ADD we can compute the probability of the node  $P(N)$  by the following expression:

$$\begin{aligned} P\{N\} &= p_1 P\{N_{succ-\ell}\} + (1 - p_1) P\{N_{succ-r}\} \\ &= P\{N_{succ-r}\} + p_1 (P\{N_{succ-\ell}\} - P\{N_{succ-r}\}) \end{aligned} \quad (1)$$

where  $N_{succ-\ell}$  is the left successor node of node  $N$  in the ADD obtained by setting  $x_i = 1$  and  $N_{succ-r}$  is the right successor of node  $N$  obtained by setting  $x_i = 0$ . Starting from a terminal leaf of the ADD where the reward function  $\Psi_{s,t}$  has a value  $\psi$  we apply recursively Equation 1 backward up to the root node and we sum up at each node the corresponding probability value along the path. The obtained value at the root node is the probability that the  $\Psi_{s,t}$  function is equal to  $\psi$ .

**Example 1: Weighted bridge network with cost** - A bridge network with weighted directed arcs is shown in Figure 6. In this first example, we assume that the labels on the edges represent the length of the arcs. Furthermore, a failure probability equal to  $p = 0.9$  is uniformly assigned to all the arcs. Referring to *Problem 1*, we want to compute the probability to reach node  $t = 4$  from  $s = 1$  with a distance below a given threshold.

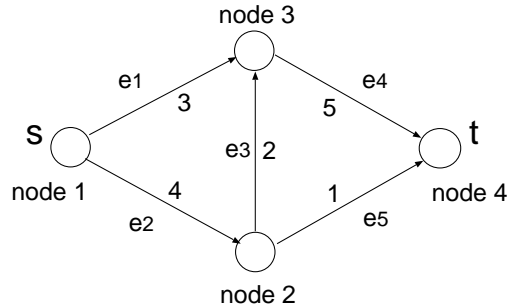


Figure 6: A directed bridge network

It is clear from a simple visual inspection of the network of Figure 6, that there are three minpaths of length, respectively:

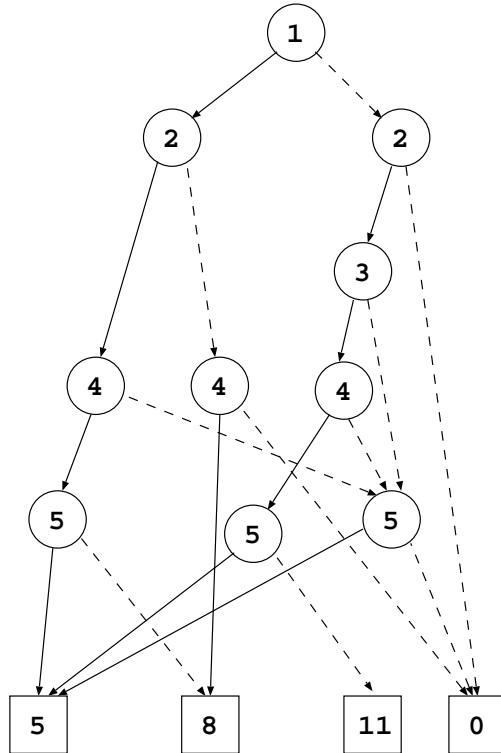


Figure 7: ADD bridge network, with weights interpreted as lengths

$$\begin{aligned}
 H_1 &= e_1 e_4 & H_2 &= e_2 e_5 & H_3 &= e_2 e_3 e_4 \\
 l(H_1) &= 8 & l(H_2) &= 5 & l(H_3) &= 11
 \end{aligned}
 \tag{2}$$

Assuming a maximum threshold greater than the maximum length, as for instance  $\psi_{max} = 15$ , all the minpaths satisfy the constraint and the corresponding ADD is shown in Figure 7. The probability values of the different ADD terminal values are computed according to Equation (1) and are reported in Table 2. The terminal value 0 gives the probability that the constraint is not satisfied (in this case,  $s - t$  are not connected).

If we change the maximum accepted distance and we fix  $\psi_{max} = 10$ , we obtain the results shown in Table 3. The probability for the terminal value 0 in Table 3 is the sum of the probabilities that  $s - t$  are not connected, or they are connected with a distance greater than  $\psi_{max} = 10$ . This probability is the sum of the last two rows of Table 2.

**Example 2: Weighted bridge network with capacity** - We consider the same network of Figure 6 with the same labels. But in this example the arc labels are interpreted as the capacities of the link, and we

Table 2: Probability of the ADD terminal values (distance) for a threshold  $\psi_{max} = 15$

(s-t) Distance	Probability
5	0.81
8	0.1539
11	0.00729
0	0.02881

Table 3: Probability of the ADD terminal values (distance) for a threshold  $\psi_{max} = 10$

(s-t) Distance	Probability
5	0.81
8	0.1539
0	0.0361

apply *Problem 2*. With a minimum threshold  $\psi_{min} = 1$ , we obtain the ADD displayed in Figure 8.

The values on the terminal leaves of the ADD represent the maximum flow that the network can carry along a path starting from the root node. For example:

- The terminal leaf 6 is reached along the path 1, 2, 3, 4, 5 representing that all the arcs are up. In this case the maximum flow is carried by the weakest cut 4, 5.
- The terminal leaf 3 is reached along the paths  $(1, 2, \bar{3}, 4, \bar{5})$  or  $(1, \bar{2}, 3, 4)$  or  $(1, \bar{2}, \bar{3}, 4)$  where the weakest cut is given by the arc 1 only, that carries a flow of 3. But another possible path is  $(\bar{1}, 2, 3, 4, 5)$  where the weakest cut is  $(3, 5)$  that carries also a maximum flow equal to the sum of the capacities of arcs 3 and 5: i.e.  $2 + 1 = 3$ .
- The terminal leaf 1 can be reached along many paths, as for instance  $(\bar{1}, 2, 3, \bar{4}, 5)$ , that are all characterized by the fact that the weakest link is arc 5 that carries a capacity of 1.

As in the previous example a failure probability equal to  $p = 0.9$  is uniformly assigned to all the arcs. The values of the probability for the different terminal values of the ADD of Figure 8, are reported in Table 4.

By assuming a minimum threshold  $\psi_{min} = 4$  the probability values are reported in Table 5.

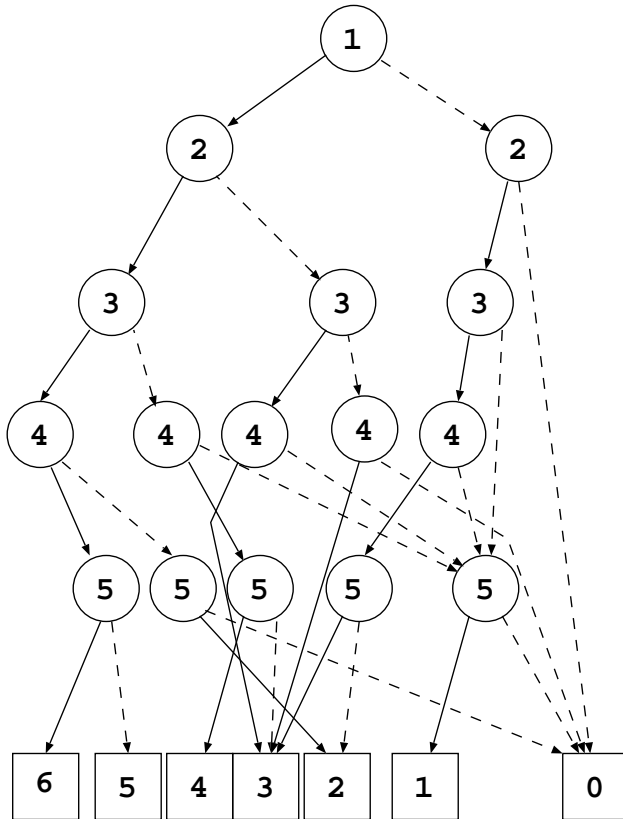


Figure 8: ADD bridge network, with weights interpreted as capacities

## 5 Service Availability of the GARR Network

With reference to Figure 1, we have assigned possible weights to the arcs of the network in two different ways:

- Weights are the bandwidths as derived from the legend of Figure 1.
- Weights are the geographical distances between any two nodes. The distances are measured in kilometers from a corresponding geographical map. In this case, we make the assumption that the cost of the connection between any two nodes is proportional to the length of the connection itself.

As in the previous case we consider two couples of source and sink nodes:  $TO - CT$  and  $TS1 - NA$ . Furthermore, we assume that all the links have a probability  $p = 0.9$  of being up and a probability  $1 - p = 0.1$  of being down. The network is considered as non directed so that the various links can be traversed in both directions.

Table 4: Probability of the ADD terminal values (max flow) for a threshold  $\psi_{min} = 1$

(s-t) Max Flow	Probability
6	0.59049
1	0.08829
5	0.06561
4	0.06561
3	0.1539
2	0.00729
0	0.02881

Table 5: Probability of the ADD terminal values (max flow) for a threshold  $\psi_{min} = 4$

(s-t) Max Flow	Probability
6	0.59049
5	0.06561
4	0.06561
0	0.27829

### 5.1 Weighted GARR network: Bandwidth availability

Resorting to the GARR Web site the bandwidth of the major network connections can be obtained. In this way, we can compute the probability of assuring a given capacity level between any two nodes. If we don't fix a minimum bandwidth, the analysis tool provides all the capacities that can be established between the source and the sink node. For the connection  $TO - CT$  we have assumed a minimum threshold  $\psi_{min} = 2500 Mbps$ , so that only the connections that carry a capacity greater than the minimum value  $\psi_{min}$  are reported in Table 6. The table is ordered from the maximum achievable bandwidth (5000 Mbps) to the minimal capacity exceeding  $\psi_{min} = 2500$ . The row with zero entry corresponds to the probability that the two nodes are not connected or are connected with a capacity lower than the fixed minimum.

From Table 6, the probability that a capacity greater than  $\psi_{min} = 2500 Mbps$  is carried between  $TO$  and  $CT$  is obtained by summing all the rows satisfying the constraint and is equal to 96% (also equal to 1 minus



Flow (Mbps)	Probability
5000	0.5795
2830	0.0014
2820	0.0006
2810	0.0002
2685	0.0069
2675	0.0055
2665	0.0036
2655	0.0096
2530	0.0027
2520	0.0020
2510	0.0201
2500	0.3292
0	0.0382

Table 6: Maximum flow and corresponding probability between  $s = TO$  and  $t = CT$

the probability of the row with zero value).

For the connection  $TS1 - NA$  we have assumed a minimum threshold  $\psi_{min} = 1$ , so that all the connecting paths are included in the analysis. The results for the connection between  $TS1$  and  $NA$  are reported in Table 7. The row corresponding to the zero value represents the probability that there is no connection between source and sink.

If we set in Table 7 a minimum threshold  $\psi_{min} = 2500 Mbps$  we get that the probability that a capacity greater than  $\psi_{min} = 2500 Mbps$  is carried between  $TS1$  and  $NA$  is equal to 87%.

## 5.2 Weighted GARR network: Distance availability

In this second experiment, we suppose that the cost of the connection is proportional to its distance. To any edge we assign a weight corresponding to the distance (in  $km$ ) between the pair of nodes and we calculate the probability to reach the sink node with a distance less than a preassigned threshold.

Assuming as source node  $TO$  and as sink node  $CT$  the obtained results are reported in Table 8. Since the minimal distance of a connection  $TO - CT$  is  $1522 Km$ , in generating Table 8 we have assumed a threshold

Flow (Mbps)	Probability
2655	0.6366
2520	0.0002
2510	0.0188
2500	0.2176
340	0.0004
330	0.0038
320	0.0017
310	0.0007
185	0.0016
175	0.0028
165	0.0015
155	0.0808
30	0.0003
20	0.0005
10	0.0060
0	0.0263

Table 7: Maximum flow and corresponding probability between  $s = TS1$  and  $t = NA$

$\psi_{max} = 1700 \text{ km}$  so that in Table 8 only the connections with a distance lower than  $\psi_{max}$  are quoted. The row with value zero reports the probability that the connection is not established or it is established with a total length greater than  $\psi_{max}$ .

To give a more detailed picture of the distance values through which the source node  $TO$  can reach the sink node  $CT$ , we have reported in Figure 9 the histogram and the corresponding empirical cumulative distribution function of the distances of all the possible paths connecting  $TO$  to  $CT$ .

We have repeated the distance computations between  $TS1$  and  $NA$ . In this case, we have assumed as a maximum acceptable distance  $\psi_{max} = 1300 \text{ km}$ . Table 9 reports the obtained values ordered vs increasing distance. The row with value zero reports the probability that the connection is not established or it is established with a total length greater than  $\psi_{max}$ .

Similarly to the previous case, we have reported in Figure 10 the histogram and the corresponding empirical cumulative distribution function of the distances of all the possible paths connecting  $TS1$  to  $NA$ .

Distance (Km)	Probability
0	4.43418e-02
1522	6.56100e-01
1525	1.24659e-01
1536	5.90490e-02
1539	1.12193e-02
1561	6.91464e-02
1564	1.60023e-02
1567	9.34540e-03
1575	6.22317e-03
1578	1.44021e-03
1581	8.41086e-04
1635	1.09153e-03
1649	9.82374e-05
1658.9	3.13806e-04
1672.9	2.82426e-05
1677	9.22750e-05
1691	8.30475e-06

Table 8: Connection distance between  $TO$  and  $CT$  lower than  $\psi_{max} = 1700 \text{ km}$

## 6 Conclusion

The NRA tool has been extended by allowing to analyze weighted stochastic networks, where weights can assume different physical meanings. To show the potentialities and the capabilities of the extended tool we have extensively examined a case study taken from a net structure available from the Web. In particular we have concentrated our attention on the Italian GARR network and we have studied the connectivity properties of the GARR network without weights (pure reliability analysis) and with added weights (service availability analysis).

Because of the lack of real data on the reliability of the elementary elements of the network (point-to-point connections) we have always arbitrarily assumed in our study a uniform probability  $p$  (usually  $p = 0.9$ ) of each connection of being up. This value is not related to the real reliability of the GARR network but could

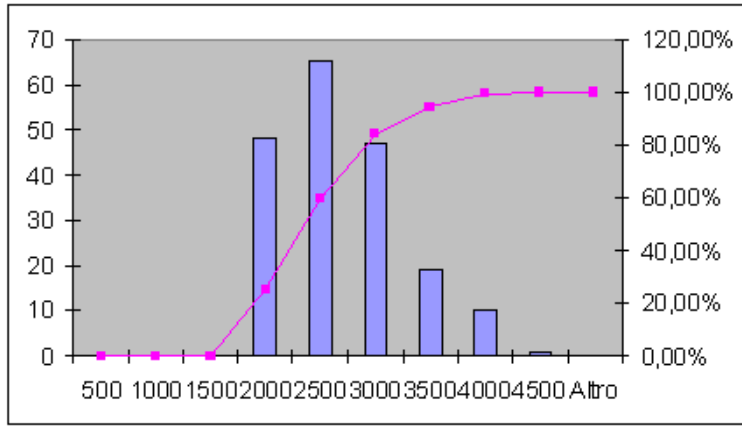


Figure 9: Histogram of the distance values for  $s = TO$ ,  $t = CT$

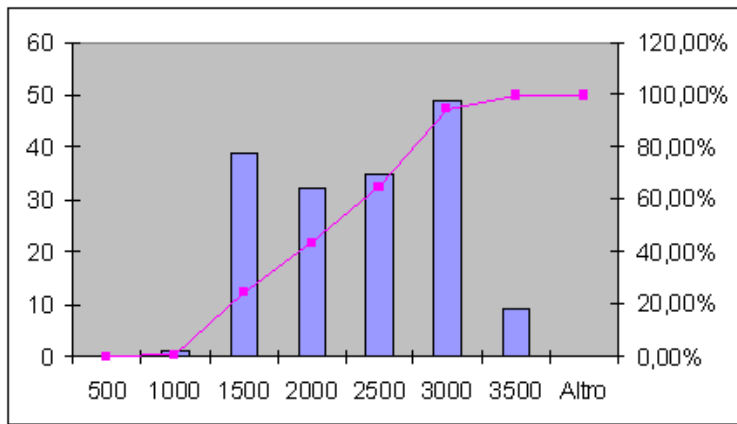


Figure 10: Histogram of the distance values for  $s = TS1$ ,  $t = NA$

be easily modified in the NRA tool in the presence of more realistic experimental evidence.

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Distance (Km)	Probability
0	6.4723e-02
923	5.9049e-01
1145	4.3046e-02
1187	3.8742e-03
1211	1.9076e-01
1229	6.3688e-02
1231	5.0046e-03
1237	4.5041e-04
1250	3.0404e-02
1253	4.5582e-03
1256	1.6864e-03
1264	1.1979e-03
1273	1.0173e-04
1279	9.1563e-06

Table 9: Connection distance between *TS1* and *NA* lower than  $\psi_{max} = 1300 \text{ km}$

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