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Renvoi in Private International Law: a Formalization with Modal Contexts

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Renvoi in Private International Law: a Formalization with Modal Contexts

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Abstract. The paper deals with the problem of formalizing the renvoi in private international law. A rule based (first-order) fragment of a multimodal logic including context modalities as well as a (simplified) notion of common knowledge is introduced. It allows context variables to occur within modalities and context names to be used as predicate arguments, providing a simple combination of meta-predicates and modal constructs. The nesting of contexts in queries is exploited in the formalization of the renvoi problem. A sound and complete proof procedure is provided.

1 Introduction

Given an international matter (is Taro a heir of John?), one wants to decide whether the matter is valid in a given country (such as in Japan) or not. In some cases, such as when Taro's parents do not have the same nationality, this matter cannot be answered only considering the legislation of one country, and requires the determination of the jurisdiction of the matter. For instance, if there is a legal child-parent relationship between Taro and John in John's home country, the application of the law in Japan, means the application of the law in force in that country.

Private international law "enables the coexistence of multiple normative systems, having distinct and often contradictory rules" [5]. Deciding the jurisdiction over a certain case, i.e. establishing which country has the jurisdiction over that case, is only one of the different tasks which have to be considered for modeling private international law, and Dung and Sartor in [5] also consider the issue of deciding the court having competence as well as the issue of establishing the legal system according to which the court has to decide. Dung and Sartor provide an analysis of private international law and propose a formal model based on modular argumentation.

In this paper, we specifically consider the so-called *renvoi*: determining the jurisdiction in one country may require for the determination of the jurisdiction in another country, a situation which may generate a sequence of references to different countries. Renvoi is not considered in [5]. As for the work in [5], our work is not intended to deal with normative conflicts, as done in the belief revision approaches, starting with the seminal work in [4], and in the defeasible reasoning approaches to normative conflicts [12, 11, 9], which usually require some kind of priority among norms to be taken into account. As observed by Dung and Sartor, private international law enables the coexistence of multiple normative systems having contradictory rules without the necessity of defining priorities among the rules or systems: “conflicts between competences and between rules are avoided by distributing the cases between authorities of the different normative systems (jurisdiction) and by establishing what set of norms these authorities have to apply to each given case (choice of law)”. There are only limited exceptions to this principle. This motivates our choice of dealing with scenarios, as the one introduced below, using a monotonic modal formalism with contexts, although, in the general case, a nonmonotonic formalism might be needed (and Dung and Sartor base their logical model on modular argumentation).

Let us consider the following scenario. For simplicity, we do not consider the competence issue and assume the legal system of the country of jurisdiction is always applied.

Example 1 (Renvoi). Suppose the following laws hold in *every country*:

1. Inheritance matter, such as a property of heir, will be determined in jurisdiction of the home country of Descendant.
2. A legitimate child-parent relationship between Child and Parent will be determined in jurisdiction of the home country of Parent, or determined in jurisdiction of the home country of Spouse of Parent if there is a biological child-parent relationship between Child and Parent.
3. Marriage will be determined in jurisdiction of the home country of either spouse.
4. The home country is Person’s nationality, if Person has only one nationality.
5. The home country is decided by the most related country for a Person, if Person has multiple nationality.
6. The most related country for Person is usually the country of Person’s habitual residence.

Domestic Rules that hold in *Japan*:

1. A marriage relationship holds between Spouse 1 and Spouse 2 if there is an agreement on marriage between Spouse1 and Spouse 2 and they register their marriage in Japan.
2. Child is a heir of a Parent if there is a child-parent relationship between them.
3. Child and Parent have a child-parent relationship if there is a legitimate child-parent relationship between them, or if there is a non-legitimate child-parent relationship between them.

Domestic Rules that hold in *Coutry1*:

1. A marriage relationship holds between Spouse 1 and Spouse 2 if there is an agreement on marriage between Spouse1 and Spouse 2 and they register their marriage in Coutnry1.
2. Child is a heir of a Parent if there is a legitimate child-parent relationship between them.

Furthermore, we have the following facts:
- John has multiple nationalities of Country1 and Country2.
- Yoko has a single nationality of Japan.
- John usually lives in Coutry1.
- John and Yoko agreed to get married and registered their marriage at Coutry1.
- John and Yoko had a son named Taro.

Consider the following questions:
- 'John is married with Yoko' is valid in Japan?
- 'Yoko is married with John' is valid in Japan?
- 'Taro is a heir of John' is valid in Japan?
- 'Taro is a heir of Yoko' is valid in Japan?

Motivated by the scenario above, in this paper we introduce a formalism which is the rule-based (first order) fragment of a multimodal logic including context modalities as well as a (simplified) notion of common knowledge. For instance, in the example above, legislation of Japan can be represented by a modal context while general laws (such as the jurisdiction laws), which hold in any context, exploit context variables and global facts are captured as (common) knowledge. In the simplified example we are considering, we assume a single set of jurisdiction rules rather than one for each country.

As we will see, the formalism allows the interactions among contexts to be captured, context variables to occur within modalities and context names to be used as predicate arguments, thus supporting a simple combination of meta-predicates and modal constructs. A variant of this modal language, which does not admit context variables but includes hypothetical implications, was developed in [1]. While the completeness proof for the calculus in [1] directly exploits the sequent calculus for the modal logic, a direct completeness proof is possible for this rule language, using a canonical model construction.

The structure of the paper is the following. In the next section, we introduce the rule-based fragment with modal contexts. The semantics and the proof procedure of the rule-based fragment are studied, respectively, in Section 3 and 4, which also contains the soundness and completeness result. In Section 5 we reconsider the initial formulation of the running example and refine it to deal with the renvoi problem, by combining meta-predicates and modal constructs. Section 6 concludes the paper.

2 A modal formalization

We consider the rule-based fragment of the language in [1], extended by allowing variables to occur within modalities in rule definitions. Let us $\mathcal{L}_k^{\Box}$ be a first order multimodal language containing: countably many variables, constants, function and predicate symbols; a finite set $Ctx = \{c_1, \ldots, c_n\}$ of constant symbols, called contexts; the logical connectives $\neg$, $\wedge$, $\supset$, and quantifiers $\forall$ and $\exists$, as in the predicate calculus, and the modalities $\Box$ and $[C]$, where C can be a variable or a context constant c_i in Ctx.

As the variables X occurring in a modality $[X]$ are intended to be instantiated only with constants in Ctx (as we will see later), the ground formulas of the language may contain two kinds of modalities: the modalities $[c_1], \ldots, [c_k]$, which represent k different contexts and the modality $\Box$, which can be regarded as a sort of (weak) "common knowledge" operator. A modal formula $[c_i]\alpha$ can be read as "α belongs to context c_i" or "agent c_i believes α". A modal formula $\Box\alpha$ can be read as "α holds in all contexts" or "all agents believe α".

Let A represent atomic formulas of the form $p(t_1, \ldots, t_s)$, where p a predicate symbol and $t_1, \ldots, t_s$ are terms of $\mathcal{L}$, and let $\top$ be a distinguished proposition (*true*). The syntax of the *clausal fragment* of $\mathcal{L}_k^\Box$ is the following:

$$G ::= \top \mid A \mid G_1 \wedge G_2 \mid \exists x G \mid [a_i]G \mid [X]D \mid \Box G$$
$$D ::= H \leftarrow G \mid D_1 \wedge D_2 \mid [c_i]D \mid [X]D \mid \Box D \mid \forall x D$$
$$H ::= A \mid [c_i]H \mid [X]D \mid \Box H$$

where G stands for a *goal*, D for a *clause* or *rule*, H for a *clause head*. Sequences of modalities may occur in front of goals, in front of rule heads and in front of rules. In the following D will interchangeably be regarded as a conjunction or a set of clauses (rules). A program P consists of a closed set of rules D. Also, we will adopt the convention that all the variables free in a rule D are implicitly universally quantified in front of it.

We say that a program P is *context safe* if each variable X occurring in a modality $[X]$ in a rule D of P, also occurs in an atom $context(X)$ in the body of D. We assume the predicate $context$ has a built_in definition as $\forall X(context(X) \leftrightarrow (X = c_1 \vee \ldots \vee X = c_k))$, so that the context safeness condition guarantees that each context variable will be bounded to some context constant in all the possible groundings of the program P. In essence, this corresponds to a typing condition.

Referring to the example above, we can introduce the contexts `japan` and `country1` containing, respectively, the domestic rules specific to `japan` and to `country1`, using a Prolog-like notation, as follows:

```
□[japan] {
    heir(Child, Parent) :−child_parent_rel(Child, Parent).
    child_parent_rel(Child, Parent) :−
        legitimate_child_parent_rel(Child, Parent).
    child_parent_rel(Child, Parent) :−
        non_legitimate_child_parent_rel(Child, Parent).
    marriage(Spouse1, Spouse2) :− agreement(marriage, Spouse1, Spouse2),
        registered(marriage, Spouse1, Spouse2, japan). }

□[country1] {
    heir(Child, Parent) :−legitimate_child_parent_rel(Child, Parent).
    child_parent_rel(Child, Parent) :−
        legitimate_child_parent_rel(Child, Parent).
    legitimate_child_parent_rel(Child, Parent) :−
        marriage(Parent, S), biological_child_parent_rel(Child, Parent).
    marriage(Spouse1, Spouse2) :− agreement(marriage, Spouse1, Spouse2),
        registered(marriage, Spouse1, Spouse2, japan). }
```

The modality $\Box$ in front of the context modalities [`japan`] and [`country1`] is needed to make each context definition globally visible from all the other contexts (so that a goal $[c_i]G$ can occur in the body of any, local or global, rule in the program). Observe that non-modal atoms in the body of rules in a context (such as `marriage(Parent, S)` in the third rule of context `country1`) can be proved either locally to the same context (i.e., using a rule in `country1`) or using other rule definitions as those introduced below.

The following rules establish the validity of a property in some country, based on properties which may hold in the same or other countries (or globally). They are intended to capture laws (1), (2) and (3). The modalities [CountryA] and [CountryB] can only be instantiated with the constants japan, country1 and country2:

(A) □[CountryA](heir(Child, Parent) :–
  context(CountryA), context(CountryB),
  home_country(Parent, CountryB)), [CountryB]heir(Child, Parent)).

(B) □[CountryA](legitimate_child_parent_rel(Child, Parent) :–
  context(CountryA), context(CountryB), home_country(Parent, CountryB),
  [CountryB]legitimate_child_parent_rel(Child, Parent)).

(C) □[CountryA](legitimate_child_parent_rel(Child, Parent) :–
  [CountryA]marriage(Parent, Spouse), home_country(Parent, CountryB),
  [CountryB]legitimate_child_parent_rel(Child, Parent),
  biological_child_parent_rel(Child, Parent)).

(D) □[CountryA](marriage(Spouse1, Spouse2) :–
  home_country(Spouse1, CountryB),[CountryB]marriage(Spouse1, Spouse2)).

For instance, the second rule states that a legitimate child-parent relationship holds in CountryA if it holds in CountryB, where CountryB is the home country of the parent.

Global rules and facts can be encoded prefixing them with the □ operator, to mean that they are visible anywhere in the program (including contexts japan and country1):

□(marriage(Spouse1, Spouse2) :– marriage(Spouse2, Spouse1)).
□(home_country(Person, Country) :– single_nationality(Person, Country)).
□(home_country(Person, Country) :–
  multi_nationality(Person, Country), most_related(Person, List, Country)).
□(most_related(Person, List, Country) :–
  habitual_residence(Person, Country), member(Country, List)).
□ multi_nationality(john, [country1, country2])).
□ habitual_residence(john, country1)).
□ single_nationality(yoko, japan)).
□ bilogical_child_parent_relation(taro, john)).
□ bilogical_child_parent_relation(taro, yoko)).
□ agreement(marriage, john, yoko)). □ registering(marriage, john, yoko, coutry1)).
□ agreement(marriage, yoko, john)). □ registering(marriage, yoko, john, coutry1)).

3 Semantics

The semantic of the language $\mathcal{L}_k^\Box$ in [1] is a first order normal polymodal logic semantics, based on Kripke interpretations in which domains of worlds are increasing, each (ground) modality $[c_i]$ is associated with an accessibility relation R_i, and the □ modality is associated with a reflexive and transitive accessibility relation Π, such that $(\bigcup_{i=0}^{k} R_i) \subseteq \Pi$.

In the following, due to space limitations, we restrict our consideration to Kripke interpretations over the Herbrand Universe for $\mathcal{L}_k^\Box$, which will be used in the following to prove the soundness and completeness of the proof procedure for our language. This restriction, however, can be done without loss of generality[1].

Definition 1. *Let P be a program of $\mathcal{L}_k^{\Box}$. A Kripke interpretation on the Herbrand universe of P is a Kripke interpretation $\mathcal{M} = \langle W, R_1, \ldots, R_k, \Pi, U_P, V\rangle$ where:*

- *W is a nonempty set of worlds;*
- *for each $i = 1, \ldots, k$, R_i is a binary relation on W;*
- *Π is a reflexive and transitive relation such that $(\bigcup_{i=0}^{k} R_i) \subseteq \Pi$;*
- *U_P is the Herbrand Universe of P;*
- *V is an assignment function, such that:*
  - *(a) V interprets terms as usual in Herbrand interpretations; i.e., $V(t) = t \in U_P$;*
  - *(b) for each n-ary predicate symbol p and each world $w \in W$, $V(p, w) \subseteq U_P^n$, i.e. $V(p, w)$ is a set of n-tuples $\langle t_1, \ldots, t_n\rangle$ where $t_i \in U_P$, for all i.*

Let us define the usual notions of satisfiability and validity for the formulas α of $\mathcal{L}_k^{\Box}$ over the language of the program P (we write $\alpha \in \mathcal{L}_{k,P}^{\Box}$).

Definition 2. *Given a Kripke interpretation $\mathcal{M} = \langle W, R_1, \ldots, R_n, \Pi, U_P, V\rangle$, the* satisfiability *of a formula $\alpha \in \mathcal{L}_{k,P}^{\Box}$, at a world $w \in W$ (written $\mathcal{M}, w \models \alpha$) can be defined inductively as follows:*

$\mathcal{M}, w \models \top$;
$\mathcal{M}, w \models p(t_1, \ldots, t_n)$ *iff* $\langle V(t1), \ldots, V(t_n)\rangle \in V(p, w)$;
$\mathcal{M}, w \models \neg\alpha$ *iff* $\mathcal{M}, w \not\models \alpha$;
$\mathcal{M}, w \models \alpha \wedge \beta$ *iff* $\mathcal{M}, w \models \alpha$ *and* $M, w \models \beta$;
$\mathcal{M}, w \models \alpha \to \beta$ *iff* $\mathcal{M}, w \not\models \alpha$ *or* $\mathcal{M}, w \models \beta$;
$\mathcal{M}, w \models \forall x\alpha$ *iff for all* $t \in U_P$, $\mathcal{M}, w \models \alpha[t/x]$;
$\mathcal{M}, w \models \exists x\alpha$ *iff there is a* $t \in U_P$, *such that* $\mathcal{M}, w \models \alpha[t/x]$;
$\mathcal{M}, w \models [a_i]\alpha$ *iff for all* $w' \in W$ *such that* $(w, w') \in R_i$, $\mathcal{M}, w' \models \alpha$;
$\mathcal{M}, w \models \Box\alpha$ *iff for all* $w' \in W$ *such that* $(w, w') \in \Pi$, $\mathcal{M}, w' \models \alpha$.

A closed formula $\alpha \in \mathcal{L}_{k,P}^{\Box}$ is satisfiable *if there is an interpretation $\mathcal{M} = \langle W, R_1, \ldots, R_n, \Pi, U_P, V\rangle$ and a $w \in W$ such that $\mathcal{M}, w \models \alpha$. α is a* valid *formula ($\models \alpha$) if, for all interpretations $\mathcal{M} = \langle W, R_1, \ldots, R_n, \Pi, U_P, V\rangle$, for all $w \in W$, $\mathcal{M}, w \models \alpha$.*

The $\Box$ modality is a weaker version of the common knowledge operator in [10], as the accessibility relation Π associated with $\Box$ includes but is not equal to the transitive and reflexive closure of the union of R_i. In particular, the *induction axiom* for common knowledge $A \wedge \Box(A \supset [a_1]A \wedge \ldots \wedge [a_k]A) \supset \Box A$ is not valid in our Kripke semantics. A similar weaker version of common knowledge operator was also suggested in [8], assuming a fictitious knower, called *any fool* (such that what *any fool* knows is known by all other agents), and in [7], where a modal resolution method was presented.

[1] Indeed, by Corollary 3.5 in [1], when we are concerned with entailment of formulas of the form $P \to G$, where P is a program and G is a goal in $\mathcal{L}_k^{\Box}$, we can restrict our consideration to Kripke interpretations over the Herbrand Universe for $\mathcal{L}_k^{\Box}$. Here, we are considering a slightly different language, where variables can occur in modalities, but notice that any context safe program P, stands for the program P' obtained by instantiating all the clauses in P with the constants from Ctx in all the possible ways, and Corollary 3.5 holds for P'.

4 Proof procedure

Given a program P in $\mathcal{L}_k^\Box$ and a goal α, we define a goal directed proof procedure to verify whether α is derivable from P. The procedure will make use of the grounding $[P]$ of a program P. Let $[P]$ be the set of ground instances of all the clauses in the program P, where all variables X occurring in the modalities $[X]$ are instantiated with constants in Ctx. $[P]$ contains *ground* clauses of the form $\Gamma_b(G \supset \Gamma_h A)$, where Γ_b and Γ_h are arbitrary sequences of modalities.

In the following we define the operational derivability of a closed goal G (i.e., a goal which does not contain free variables) from a *modal context*, namely, a sequence $L_0 L_1 \ldots L_n$ of modalities $[c_i]$ and $\Box$. The definition exploits a notion of *matching relation* among sequences of modalities, which will be defined below.

Definition 3 (Proof Procedure). *For a given program P, the operational derivability of a closed goal G from a modal context $L_0 L_1 \ldots L_n$ is defined by induction on the structure of G as follows:*

1. $L_1 \ldots L_n \vdash T$;
2. $L_1 \ldots L_n \vdash A$ *if there is a clause* $\Gamma_b(G \supset \Gamma_h A) \in [P]$ *and a* k, $k \leq n$, *such that:*
   (a) Γ_b *matches* $\Gamma_1 = L_1 \ldots L_k$,
   (b) Γ_h *matches* $\Gamma_2 = L_{k+1} \ldots L_n$, *and*
   (c) $L_1 \ldots L_k \vdash G$;
3. $L_1 \ldots L_n \vdash G_1 \wedge G_2$ *if* $L_1 \ldots L_n \vdash G_1$ *and* $L_1 \ldots L_n \vdash G_2$;
4. $L_1 \ldots L_n \vdash \exists x G$ *if* $L_1 \ldots L_n \vdash [x/t]G$, *for some closed term* t;
5. $L_1 \ldots L_n \vdash [c_i]G$ *if* $L_1 \ldots L_n[c_i] \vdash G$;
6. $L_1 \ldots L_n \vdash \Box G$ *if* $L_1 \ldots L_n \Box \vdash G$.

Given a program P and a closed goal G, we say that G is provable from P if $\epsilon \vdash G$ can be derived by applying rules 1-5 above (where ϵ is the empty context).

To verify that a clause in $[P]$ is applicable in the current context, it must be checked whether the modal operators in the clause (both in front of it and in front of its head) *match* the current context, from L_1 to L_n. In particular, the sequence of the modal operators Γ_b in front of the selected clause must match a *prefix* of the sequence $L_1| \ldots |L_n$, while the sequence of the modal operators Γ_h in front of the head of the selected clause must match the *remaining part* of the sequence. We say that a sequence Γ of modal operators matches another sequence Γ' if each modal operator in Γ matches a subsequence of operators in Γ' in the ordering. More precisely, each modal operator $[c_i]$ in Γ may only match the operator $[c_i]$, while each operator $\Box$ in Γ may match either an empty sequence or an arbitrary subsequence of modalities in Γ'.

Definition 4 (Matching relation). *Let $\Gamma = L_1 \ldots L_r$ and Γ' be two sequences of modalities, then we say that Γ* matches *Γ' if: either Γ and Γ' are empty, or $\Gamma = L_1 \ldots L_r$ (for $r \geq 1$) and there are r functions $f_1, \ldots, f_r$ such that, for all $j = 1, \ldots, r$:*

- $f_j([c_i]) = [c_i]$ *for all modal operators* $[c_i]$;
- $f_j(\Box)$ *is any sequence of modalities (including the empty one);*

and $f_1(L_1) \ldots f_r(L_r) = \Gamma'$.

It easy to see that the *matching relation* is *reflexive* and *transitive*.

Soundness of the proof procedure above with respect to the Kripke semantics can be proved for context safe programs follows form the following Lemma (whose proof is by induction on the structure of the goal G):

Lemma 1. *If* $L_1 \dots L_n \vdash G$ *then* $\models [P] \rightarrow [L_1] \dots [L_n]G$

The completeness proof for the language in [1] was based on a correspondence between goal directed proofs and specific proofs in a modal sequent calculus. Here, instead, completeness of the proof procedure has a direct proof through a canonical model construction, omitted for lack of space, leading to the following result:

Proposition 1. *For a context safe program P and goal G, $\mathcal{M} \models [P] \rightarrow G$ holds for all Kripke interpretations $\mathcal{M}$ if and only if G is derivable from $[P]$, i.e., $\epsilon \vdash G$.*

Let us first consider, as an example, the query "is Taro a heir of John valid in Japan?", which is captured by the goal $[\texttt{japan}]\texttt{heir}(\texttt{taro}, \texttt{john})$. This goal succeeds from the program above, using the following instance of rule (B):

$$\begin{aligned}&\Box([\texttt{japan}]\texttt{legitimate\_child\_parent\_rel}(\texttt{taro}, \texttt{john}) :- \\ &\quad \texttt{context}(\texttt{japan}), \texttt{context}(\texttt{country1}), \texttt{home\_country}(\texttt{john}, \texttt{country1}), \\ &\quad [\texttt{country1}]\texttt{legitimate\_child\_parent\_rel}(\texttt{taro}, \texttt{john})).\end{aligned}$$

and exploiting the definition of `heir` and `child_parent_rel` from the context `japan`, the definition of `legitimate_child_parent_rel` and `marriage` from the context `country1`, and the definition of `biological_child_parent_rel`, `agreement` and `registered` from the global facts.

5 A formalization of renvoi in private international law

The formalization of the running example given in Section 2 establishes the validity of a property in some country, based on properties which may hold in the same or other countries. For instance, in rule (A), the validity of proposition $\texttt{heir}(\texttt{Child}, \texttt{Parent})$ in the context `CountryA`, depends on the validity of the same property in context `CountryB`. However, the rules in the program do not make any distinction among the validity of a property in a context and the jurisdiction of the same property in that context. Introducing such a distinction is essential to capture renvoi.

In particular, to check property $\texttt{heir}(\texttt{taro}, \texttt{john})$ in Japan, we need first to determine the jurisdiction of the property `heir`, with Japan as applying country, using rule (A), rather than using rule for `heir` in the context `japan`. Indeed, according to law (1), an inheritance matter, such as a property of heir, is to be determined in the jurisdiction of the home country of the parent. In this example, $\texttt{heir}(\texttt{taro}, \texttt{john})$ is to be determined in "country1", as "country1" is the home country of John.

We then reformulate our query as $\texttt{holds}(\texttt{heir}(\texttt{taro}, \texttt{john}), \texttt{japan})$, and we can introduce for `heir`, as for every property whose jurisdiction is to be determined, a rule:

$$\begin{aligned}&\Box(\texttt{holds}(\texttt{heir}(\texttt{Child}, \texttt{Parent}), \texttt{CountryA}) :- \\ &\quad [\texttt{CountryA}]\texttt{jurisd}(\texttt{heir}(\texttt{Child}, \texttt{Parent}), \texttt{CountryB}), \\ &\quad [\texttt{CountryB}]\texttt{heir}(\texttt{Child}, \texttt{Parent})).\end{aligned}$$

where the goal $[\mathtt{CountryA}]\mathtt{jurisd}(\mathtt{Matter},\mathtt{CountryB})$ is used to determine the jurisdiction CountryB of the Matter in CountryA i.e., the country in which the property $\mathtt{heir}(\mathtt{Child},\mathtt{Parent})$ is to be proven.

In general, to decide the jurisdiction of a matter, we first have to determine the property involved (for instance, the matter *hair* is concerned with the property *inheritance*). The jurisdiction of a matter is then given by the jurisdiction of the corresponding property. For simplicity, we will not exemplify this aspect here. We reformulate rules $(A)-(D)$ to determine the jurisdiction of some different matter.

(A) $\Box[\mathtt{CountryA}](\mathtt{jurisd}(\mathtt{heir}(\mathtt{Child},\mathtt{Parent}),\mathtt{CountryC})$:−
$\mathtt{home\_country}(\mathtt{Parent},\mathtt{CountryB})$ $[\mathtt{CountryB}]\mathtt{jurisd}(\mathtt{heir}(\mathtt{A},\mathtt{B}),\mathtt{CountryC}))$.

(B) $\Box[\mathtt{CountryA}]($
$\mathtt{jurisd}(\mathtt{legitimate\_child\_parent\_rel}(\mathtt{Child},\mathtt{Parent}),\mathtt{CountryC})$:−
$\mathtt{home\_country}(\mathtt{Parent},\mathtt{CountryB})$
$[\mathtt{CountryB}]\mathtt{jurisd}(\mathtt{legitimate\_child\_parent\_rel}(\mathtt{Child},\mathtt{Parent}),\mathtt{CountryC}))$.

(C) $\Box[\mathtt{CountryA}](\mathtt{jurisd}(\mathtt{legitimate\_child\_parent\_rel}(\mathtt{Child},\mathtt{Parent}),\mathtt{CountryD})$:−
$\mathtt{jurisd}(\mathtt{marriage}(\mathtt{Parent},\mathtt{Spouse}),\mathtt{CountryB})$,
$[\mathtt{CountryB}]\mathtt{marriage}(\mathtt{Parent},\mathtt{Spouse})$, $\mathtt{home\_country}(\mathtt{Spouse},\mathtt{CountryC})$,
$[\mathtt{CountryC}]\mathtt{jurisd}(\mathtt{legitimate\_child\_parent\_rel}(\mathtt{Child},\mathtt{Parent}),\mathtt{CountryD})$,
$\mathtt{biological\_child\_parent\_rel}(\mathtt{Child},\mathtt{Parent}))$.

(D) $\Box[\mathtt{CountryA}](\mathtt{jurisd}(\mathtt{marriage}(\mathtt{Spouse1},\mathtt{Spouse2}),\mathtt{CountryC})$:−
$\mathtt{home\_country}(\mathtt{Spouse1},\mathtt{CountryB})$,
$[\mathtt{CountryB}](\mathtt{jurisd}(\mathtt{marriage}(\mathtt{Spouse1},\mathtt{Spouse2}),\mathtt{CountryC}))$.

where we have omitted the specification of context atoms $\mathtt{context}(\mathtt{CountryA})$, $\mathtt{context}(\mathtt{CountryB})$, etc. in the body of all the rules.

The determination tool may point out that we have to decide the validity of the matter in a different jurisdiction with respect to the current one. In rule (A) the jurisdiction for the matter $\mathtt{heir}(\mathtt{Child},\mathtt{Parent})$ is determined as the country of the parent (CountryB), which may be different for the current jurisdiction (CountryA). In such a case, we need again to decide the jurisdiction according to the private international law in the new country (i.e., CountryB). This is called a “renvoi”. If a loop in the “renvoi” is detected, the jurisdiction is set to the starting country of the loop. For example, if the private international laws determines the following sequence of jurisdictions A, B, C, D, B, then we can decide the jurisdiction for the matter to be country B.

In order to deal with such a kind of loop in renvoi, we introduce the following general rule: (R) $\Box[\mathtt{CountryA}]\Box[\mathtt{CountryA}](\mathtt{jurisd}(\mathtt{Matter},\mathtt{CountryA})$:−$\top$. As a special case of (R), the rule $\Box[\mathtt{CountryA}][\mathtt{CountryA}](\mathtt{jurisd}(\mathtt{Matter},\mathtt{CountryA})$:−$\top$ also holds, meaning that, if the determination tool points out that the validity of the matter in the current jurisdiction (i.e., in CountryA), we just take the current jurisdiction as the second argument of the predicate jurisd to be returned. For instance, when applying rule (A) in case $\mathtt{CountryA}=\mathtt{CountryB}$, the second subgoal in the body of (A) (namely, $[\mathtt{CountryB}]\mathtt{jurisd}(\mathtt{heir}(\mathtt{A},\mathtt{B}),\mathtt{CountryC}))$ immediately succeeds with $\mathtt{CountryB}=\mathtt{CountryA}$ and $\mathtt{CountryC}=\mathtt{CountryA}$, as the home country of the Parent is precisely CountryA, the country in which the determination of jurisdiction was issued.

To avoid other, spurious jurisdictions to be found, a "cut" should be added in the body of rule (R), although, of course, this is a feature which cannot be captured by rule-based language above. In [13] an encoding of cut by means of and announce predicate and an integrity constraint is exemplified, based on a notion of *global abduction*. To capture the correct behavior of renvoi, avoiding spurious solutions, an extension of the formalism with abduction or with some form of default negation would be needed. This will be subject of further work.

6 Conclusions and related work

Dung and Sartor in [5] provide a logical model of private international law, based on modular argumentation, as a way of coordinating the different normative systems without imposing a hierarchical order on them. They do not consider the issue of modeling chains of references. In this paper we exploit a rule based fragment of a modal logic with agent (or context) modalities, a simplified notion of common knowledge and context variables to capture renvoi (i.e., chains of references). As we have already mentioned above, our language is monotonic. Although modeling private international law in its full generality might require a combination of both nonmonotonicity and modularity, as shown by Dung and Sartor, private international law "conflicts between competences and between rules are avoided by distributing the cases between authorities of the different normative systems [...] and by establishing what set of norms these authorities have to apply".

As for the work in [5] our work is not intended to deal with normative conflicts, as in the belief revision approaches (starting with the seminal work in [4]) or in the defeasible reasoning approaches to normative conflicts [9, 15], which may arise in static and dynamic settings and require to deal with some kind of priority among norms. Nevertheless, an extension of the formalism with abduction or default negation will be considered.

The formalism we have considered is clearly related with many context formalisms in the literature such as, besides [1], with the CKR framework [14, 2] and with multi-context systems (mMCSs) [3], which allow for the integration of heterogeneous knowledge sources, and with other formalisms for dealing with multi-agent systems in computational logic and in Answer Set Programming (we refer to [6] for a survey).

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