NEW CHALLENGES IN NETWORK RELIABILITY ANALYSIS

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Networks in human society

Many complex physical, technological, social, biological and economical systems can be represented in the form of networks.

Networks are characterized by a set of nodes (vertices) connected by directed or undirected arcs (edges).

Vertices are the entities of the system and the edges represent the relational links among the entities.

The study of networks has proved to be beneficial in the analysis of technical and biological systems as well as of social, economical, epidemic and political relations.

Network Reliability

The present paper is aimed at investigating modelling and analysis techniques for the evaluation of the reliability of systems whose structure can be described in a form of a graph.

Given that arcs and nodes are binary entities (up and down) and given the probability of the up and down state.

Connectivity – Boolean function representing the logical way in which two nodes can be reached.

Reliability – probability that two nodes are connected

Network analysis

Small scale networks

Exhaustive search algorithms

Quantitative probabilistic analysis

Toward Large scale networks

Large networks evolve dynamically

Random failures vs malicious attacks

Network representation

A network is a graph ***G* = (*V*, *E*)** where ***V*** is the set of nodes (vertices) and ***E*** the set of arcs (edges).

Qualitative analysis;

- Minimal paths
- Minimal cuts

Quantitative analysis;

- Reliability and Unreliability functions;
- Standard algorithms and BDDs (Bryant ACM 1992).

Assumptions

Arcs can be:

- undirected
- directed

Failures can be located in:

- Arcs only;
- Nodes only;
- Both arcs and nodes.

Two special nodes are defined:

- One source node O;
- One sink node Z;

Path and minpath, Cut and mincut

For a given graph ***G=(V,E)*** a path ***H*** is a subset of components, arcs and/or nodes, that guarantees the source O and sink Z to be connected if all the components of this subset are functioning.

A path is minimal (minpath) if does not exist a subset of nodes in H that is also a path.

For a given graph ***G=(V,E)*** a cut ***K*** is a subset of components, arcs and/or nodes, that disconnect the source O and sink Z if all the components of this subset are failed.

A cut is minimal (mincut) if does not exist a subset of nodes in K that is also a cut.

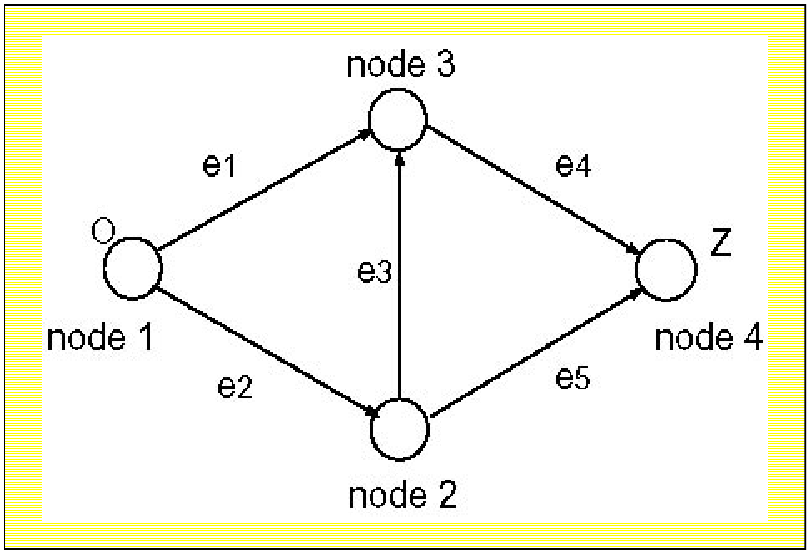
Reliability Computation

- The Reliability function of a network, can be determined from the *minpaths* :

- If $(H_1, H_2, \ldots, H_n)$ are the minpaths, then:

    - $S = H_1 \cup H_2 \cup \ldots \cup H_n$
    - $R_S = Pr\{S\} = Pr\{H_1 \cup H_2 \cup \ldots \cup H_n\}$

Example: Bridge network



List of mincut: $H1 = \{ e1, e4\}$;
$H2 = \{ e2, e3, e4 \}$;
$H3 = \{ e2, e5\}$

List of minpath: $K1 = \{ e1, e2\}$;
$K2 = \{ e2, e4 \}$;
$K3 = \{ e4, e5\}$;
$K4 = \{ e1, e3, e5 \}$

The point-to-point connectivity function can be expressed as:

$$S_{1\text{-}4} = e_1 e_4 \cup e_2 e_3 e_4 \cup e_2 e_5$$

The point-to-point reliability can be expressed as:

$$R_{1\text{-}4} = Pr\{S_{1\text{-}4}\} = p_1 p_4 + p_2 p_3 p_4 + p_2 p_5 - p_1 p_2 p_3 p_4 - p_2 p_3 p_4 p_5 - p_1 p_2 p_4 p_5 + p_1 p_2 p_3 p_4 p_5$$

Example: Bridge BDD

BDD are binary trees for
manipulating
Boolean functions
[Bryant et al. 1990]

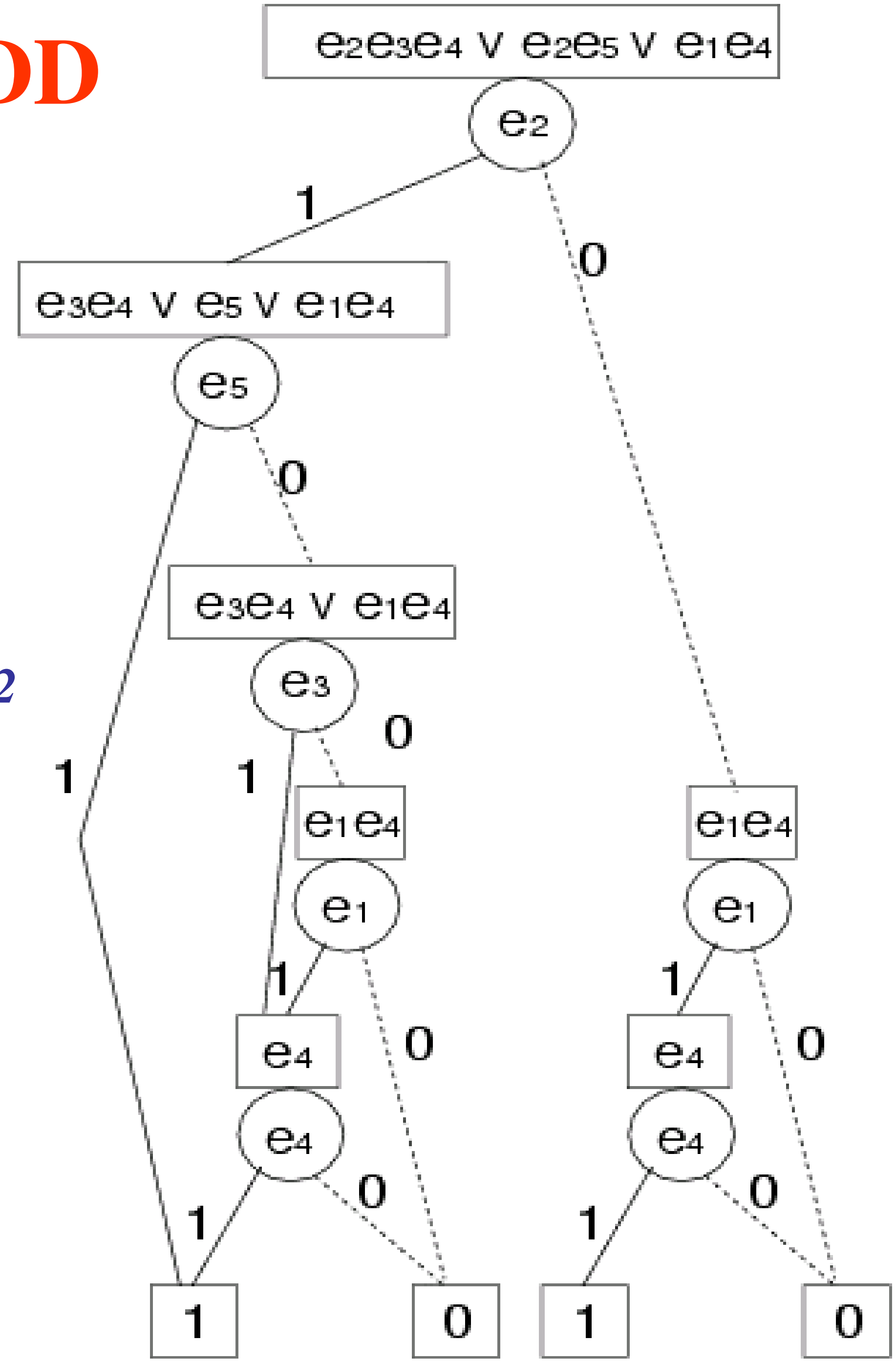
Connectivity Function:

$S_{1\text{-}4} = e_1 e_4$ ⇦ $e_2 e_3 e_4$ ⇦ $e_2 e_5$

Variables must be ordered.

Ordering

e2 Ⓞ *e5* Ⓞ *e3* Ⓞ *e1* Ⓞ *e4*



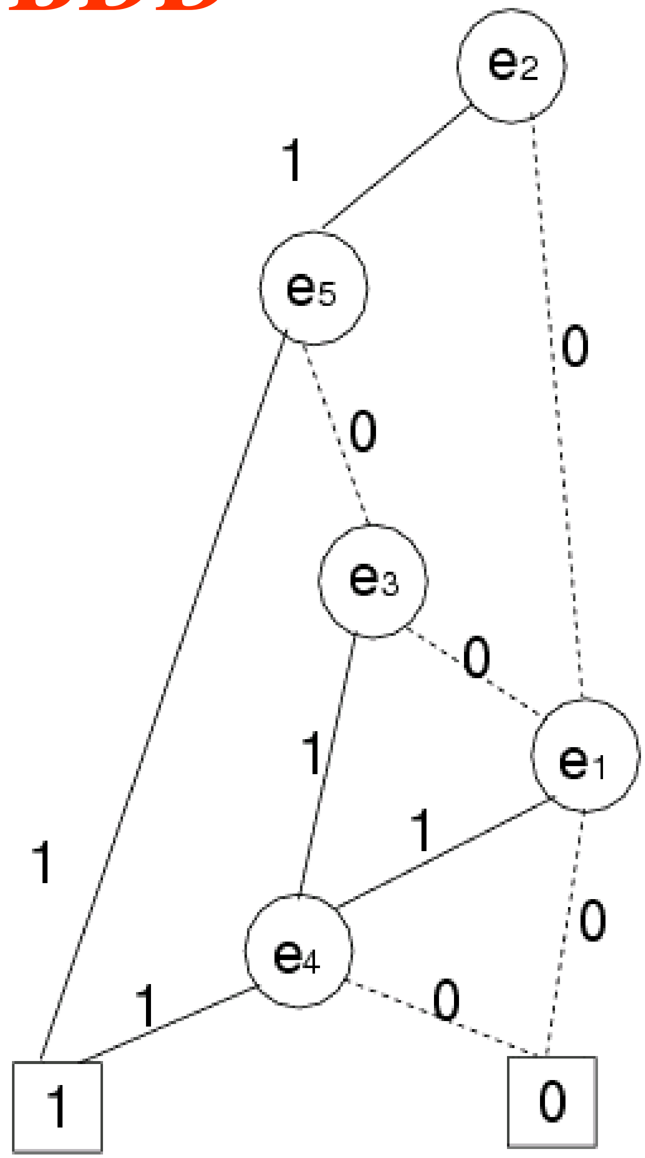
Example: Bridge ROBDD

Occasionally, the binary tree contains identical subtrees.

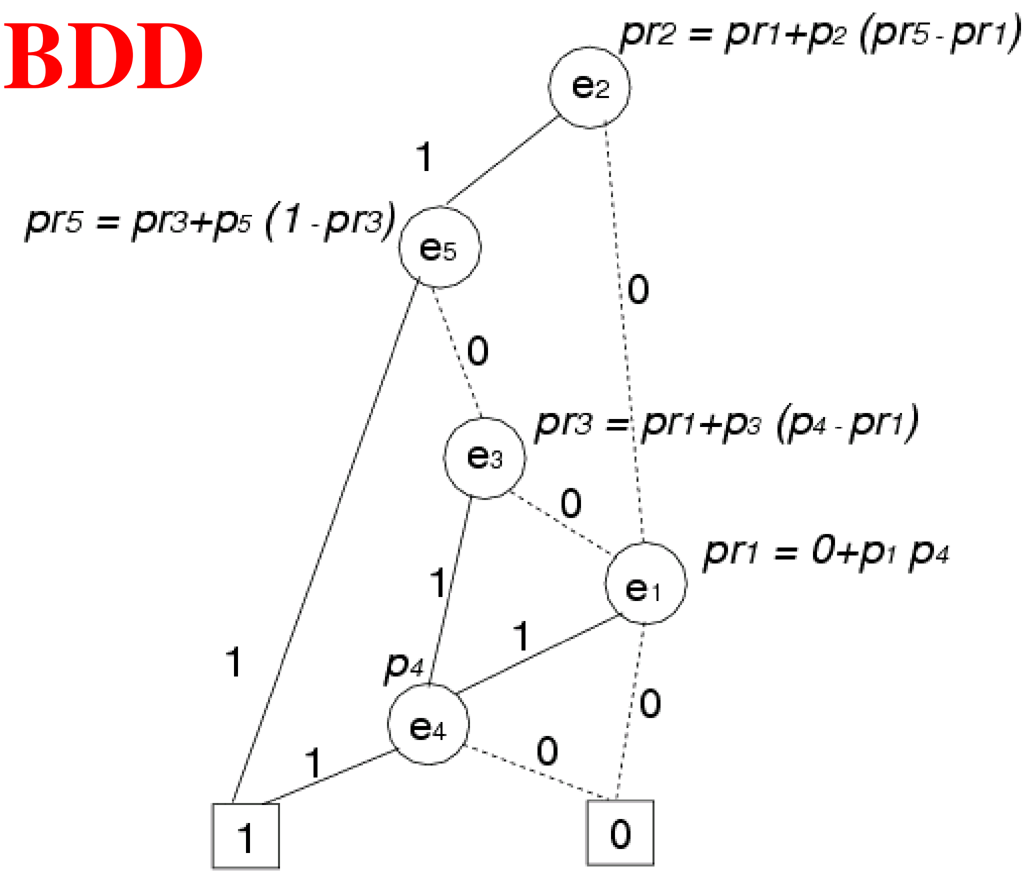
Reduction – Identical portions of BDD are folded

The subtrees at the node e_1 e_4 appear twice and can be folded.

The result is the Reduced Ordered BDD



Example: Bridge ROBDD Probability computation



The computation of the probability of the BDD proceeds recursively by resorting to the Shannon decomposition.

$$Pr\{F\} = p1 \;\; Pr\,\{F\{x1=1\}\,\} + (1 - p1)\, Pr\{\, F\{x1=0\}\,\}$$

$$= \;\; Pr\{\, F\{x1=0\}\,\} + p1\,(\, Pr\,\{F\{x1=1\}\,\} - \;\; Pr\{\, F\{x1=0\}\,\}\,)$$

Large Scale Networks

In recent years there has been an enormous amount of research work on finding general mathematical properties of large scale connected systems (networks)

Large networks evolve dynamically. Interest to study how growing networks self organize into special (scale free) structures and the role of the mechanism of preferential linking.

Two main factors

1) Network growth - the number of nodes increases throughout the lifetime of the network
2) Preferential attachment - the likelihood of connecting to a node depends on the node's degree.

Structural characteristics

1. Degree - the degree distribution of the nodes determines important global characteristics of the network.

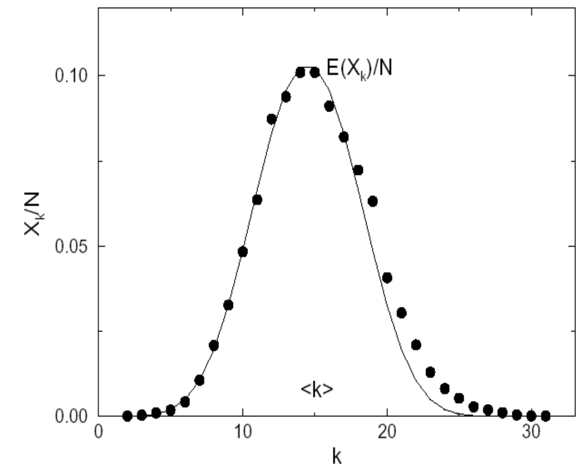
2. Clustering coefficient - Fraction of the actual number of edges present in a certain radius with respect to all the possible existing edges.

3. Shortest path - The average shortest path ❁ over all pairs of vertices is the average separation of pair of vertices.
 - lattice of dimension *d* with N vertices ❁ = N^{1/d}
 - fully connected network ❁ =1
 - if *z* is the average number of nearest neighbors *z*^ ❁ vertices are at a distance ≤ ❁ , so that N~ *z*^ ❁ .

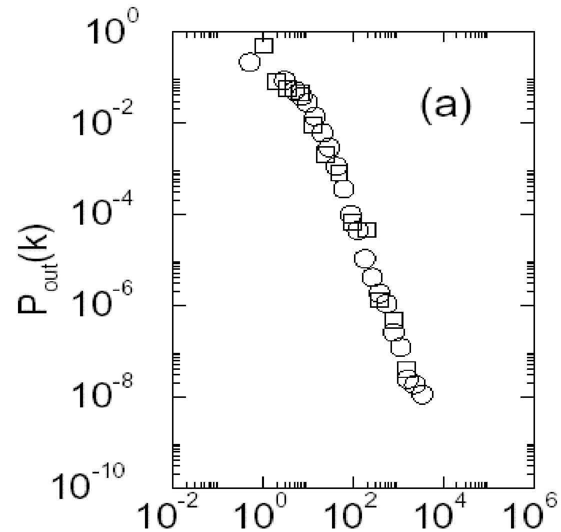
Large Scale Networks

Structure and properties of complex networks

Random Graphs – The degree distribution is Poisson;



Scale free networks - The degree distribution is power law (long tail)



The Basic Pareto Distribution

$F(x) = 1 - b / x^a$ for $x \geq b$, $a > 0$

$f(x) = a\, b^a / x^{a+1}$

Error and Attack tolerance

Random node removal

- Random graph → critical fraction *fc*
- Scale free → no critical fraction

Preferential node removal

- Scale free networks are more fragile to intentional attacks than random graphs

Spreading of diseases and diffusion of viruses

- In random networks a disease spreads in the whole network only if the infection rate is larger than a critical value;
- In scale free networks the whole network is infected at any spreading rate

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