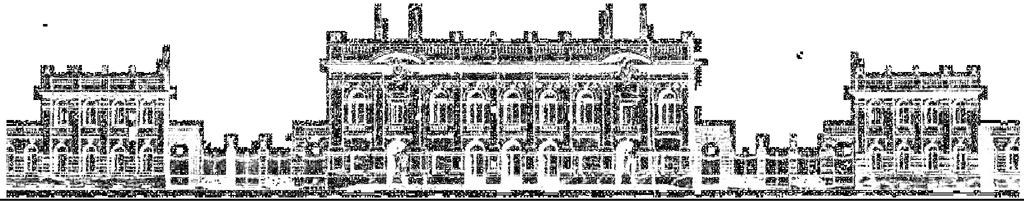

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Advanced Computational Methods for the Assessment and Optimization of Network Systems

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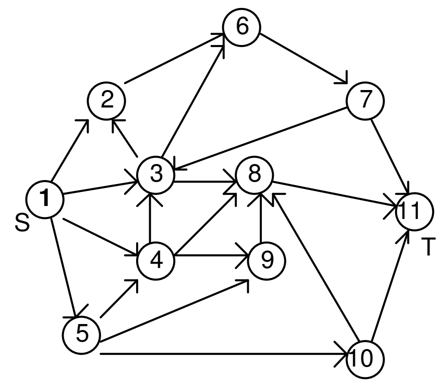
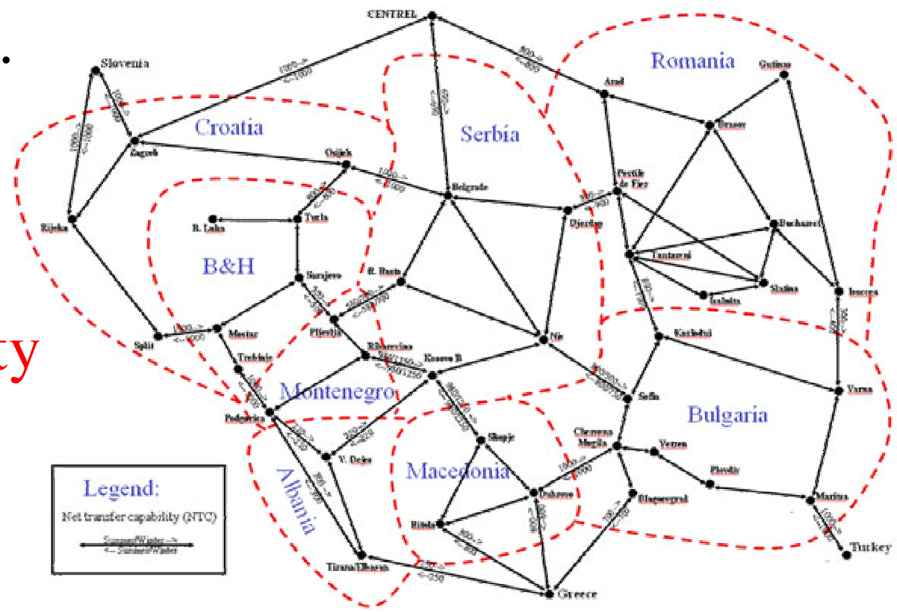
PROBLEM STATEMENT

- **WHAT?**

✓ modern society distributed services (communication, power, transportation, …)

✓ increasing reliance on technological network systems.





⇨ attention to reliability, availability, vulnerability, safety

⇨ increased system complexity

⇨ analysis by classical modelling tools difficult

NETWORK RELIABILITY

- **WHAT?**

✓ Network reliability: source-target connectivity

cut(path) sets procedure ⇨ NP-hard

- **HOW?**

✓ Cellular Automata (CA)

✓ Monte Carlo sampling and simulation (MC)

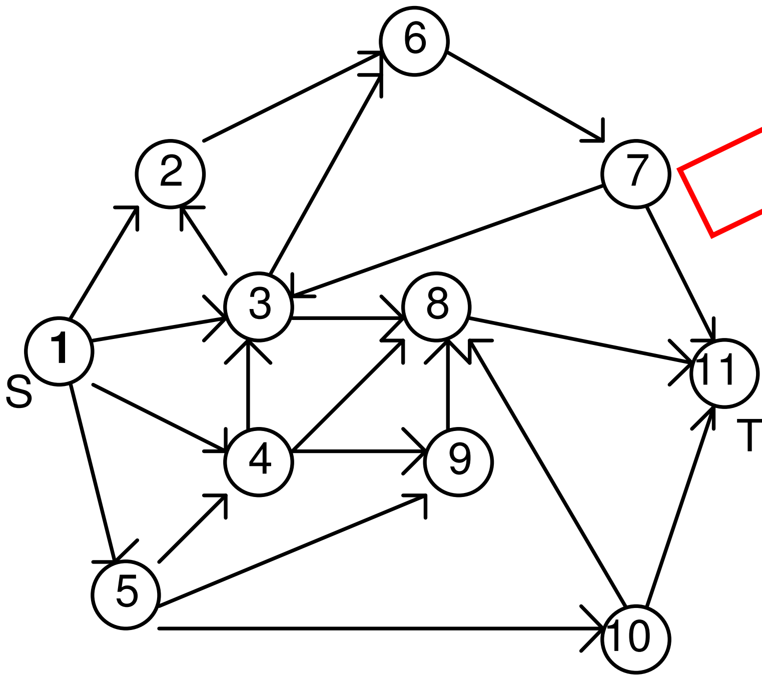
OUTLINE

- ✓ Application to network connectivity
- ✓ Application to network reliability
- ✓ Basics of MC simulation
- ✓ Application to network availability

Application to network connectivity

Application: S-T connectedness problem

(Rocco and Moreno, 2002)



1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	1	1	1	0	1	0	0

Node i = cell

N_i = set of cells which provide input to i

(e.g. $N_4 = \{1, 4, 5\}$)

s_i = 1 when node i is operating (active)

= 0 when not operating (passive)

$$s_i(t+1) = s_p(t) \vee s_q(t) \vee \ldots \vee s_r(t) \qquad p, q, \ldots, r \in N_i$$

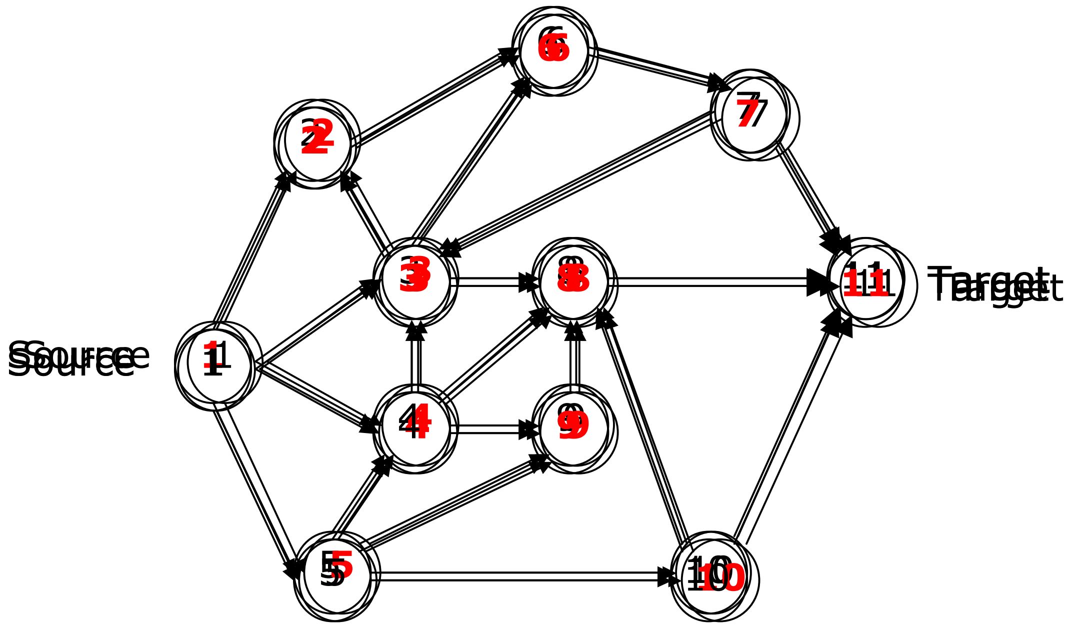
a cell is activated if there is at least one active cell in its neighbourhood

Application: S-T connectedness problem

BASIC ALGORITHM

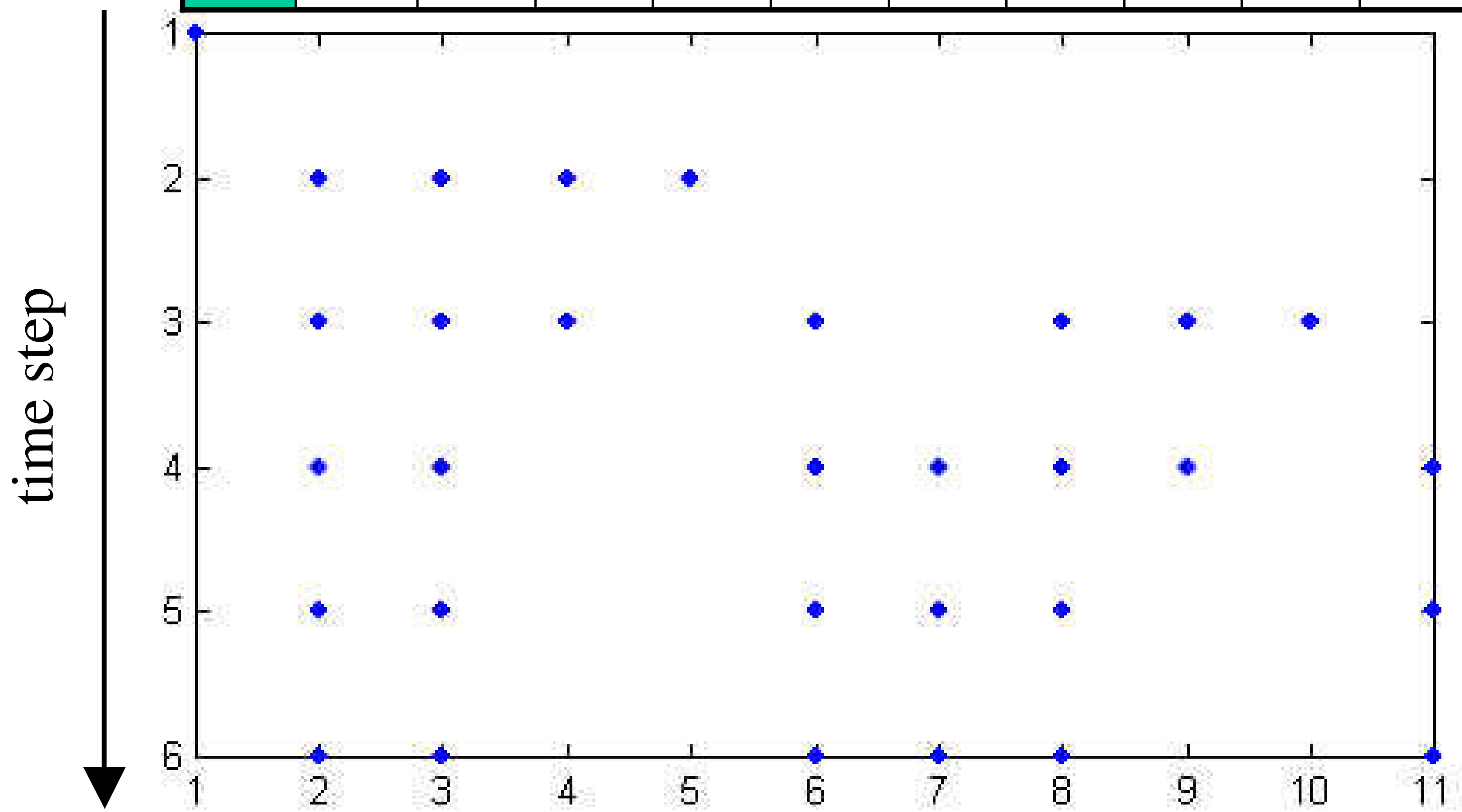
1. $t = 0$
2. Set all the cells state values to 0 (passive)
3. Set $s_s(0) = 1$ (source activated)
4. $t = t + 1$
5. Update all cells states by means of CA rule
6. If $s_T(t) = 1$, stop (target activated), else
7. If $t < m - 1$, go to 4. Else
8. $s_T(m-1) = 0$ (target passive): no connection path from S to T

Application: S-T network evolution



Application: CA evolution

1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	0	0	0	0	0	0	0



Application to network reliability

Application: S-T network reliability

(Rocco and Moreno, 2002)

w_{ji} = 1, success $\rightarrow$ p_{ji}

= 0, failure $\rightarrow$ 1-p_{ij}

$$s_i(t+1) = [s_p(t) \wedge w_{pi}] \vee [s_q(t) \wedge w_{qi}] \vee \ldots \vee [s_r(t) \wedge w_{ri}]$$

$$p, q, \ldots, r \in N_i$$

a cell is activated if there is at least one active cell in its neighbourhood and the connecting arc is functioning

Application: S-T network reliability

BASIC ALGORITHM

1. $n = 0$

2. $n = n + 1$

3. Sample by MC a realization of the states of the connecting arcs ***w***

4. Apply the previously illustrated CA algorithm for *S*-*T* connectedness, to evaluate if there is a path from *S* to *T*

5. If a path exists, then update the counter of successful system states

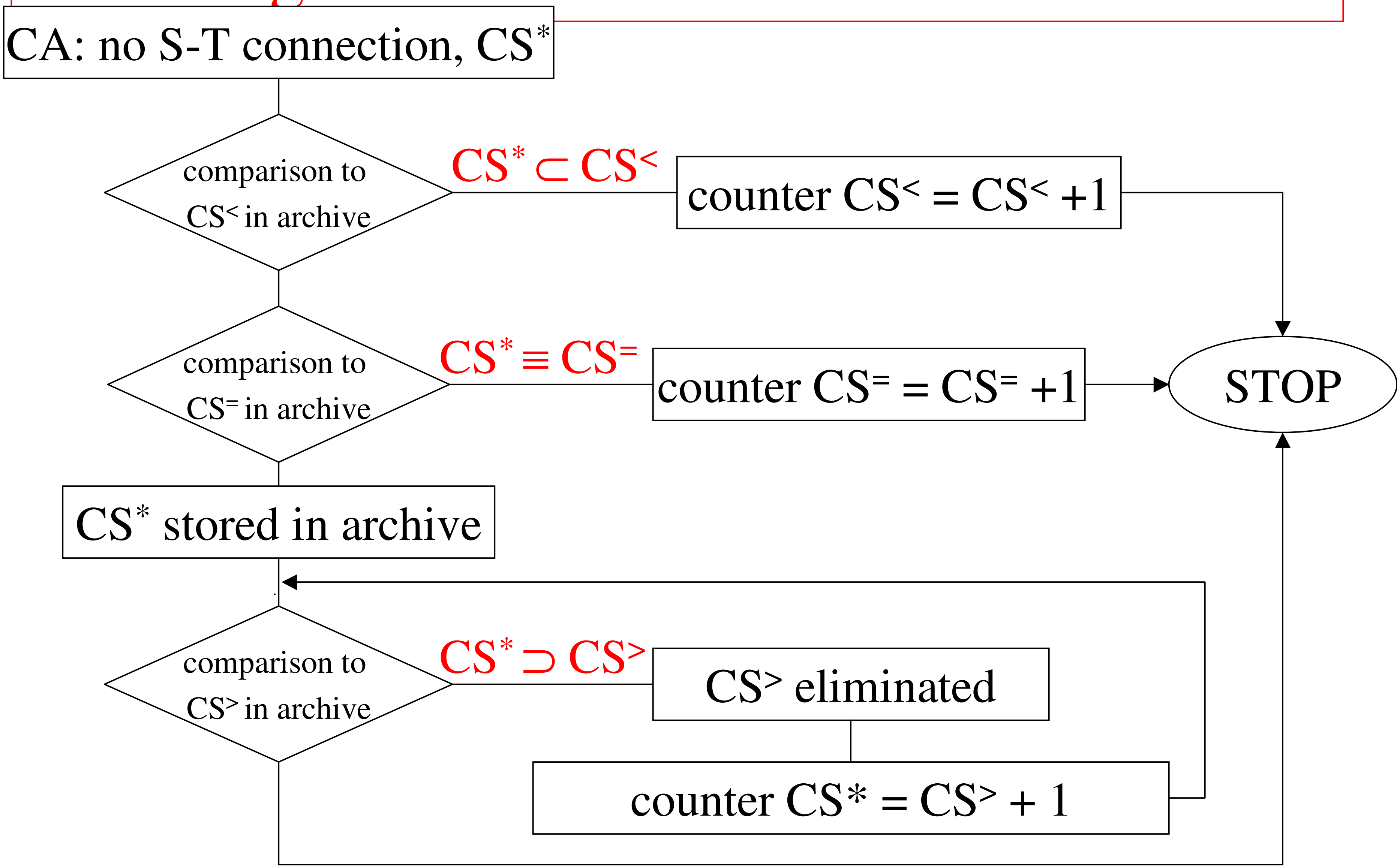
6. If MC – iteration n < N, go to 2. Else

7. Network reliability = S-T successful paths/N

Identification of minimal cut sets by CA

(Zio, Librizzi and Sansavini, 2006)

Algorithm for MCS identification



Algorithm for MCS identification

1. Keep an archive of CS

2. When a CS^* configuration is sampled
   - check whether another verified CS of $\leq$ order $\subset CS^*$ is already stored; if not
   - store the CS^* and check if higher order $\supset CS^*$

3. Update counter of occurrences for CS probability estimate

CA + MC for component importance measures

(Zio, Librizzi and Sansavini, 2006)

Algorithm for importance measure

4. Update counter of occurrences of CS containing *ji*

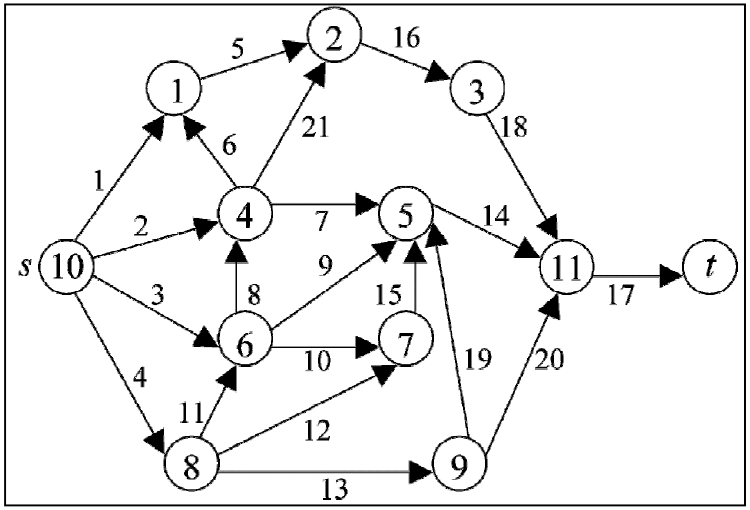
Fussell-Vesely importance measure I_{ji}^{FV}

$$I_{ji}^{FV} \approx \frac{number\ of\ occurred\ CS\ containing\ \ ji}{number\ of\ MC\ trials}$$

contribute of arc *ji* to system failure in terms of partecipation to MCS

Case study

Literature case study: MCS identification

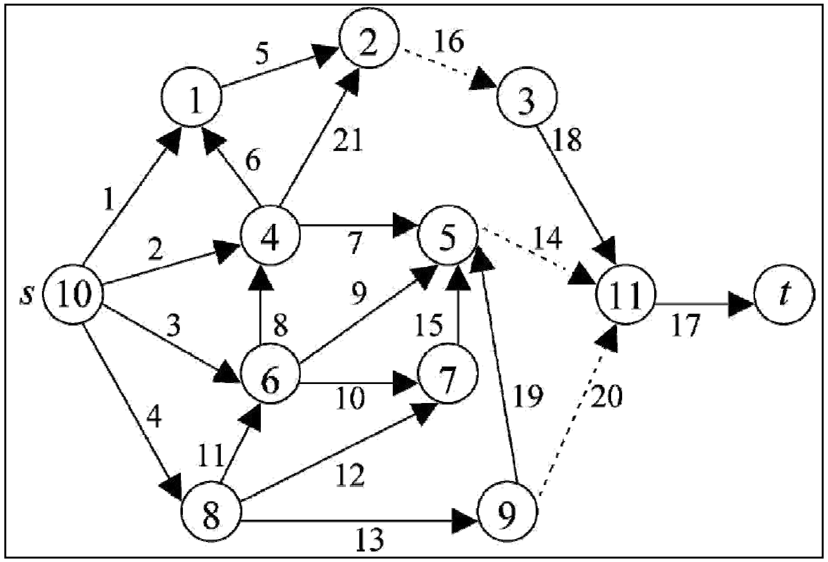


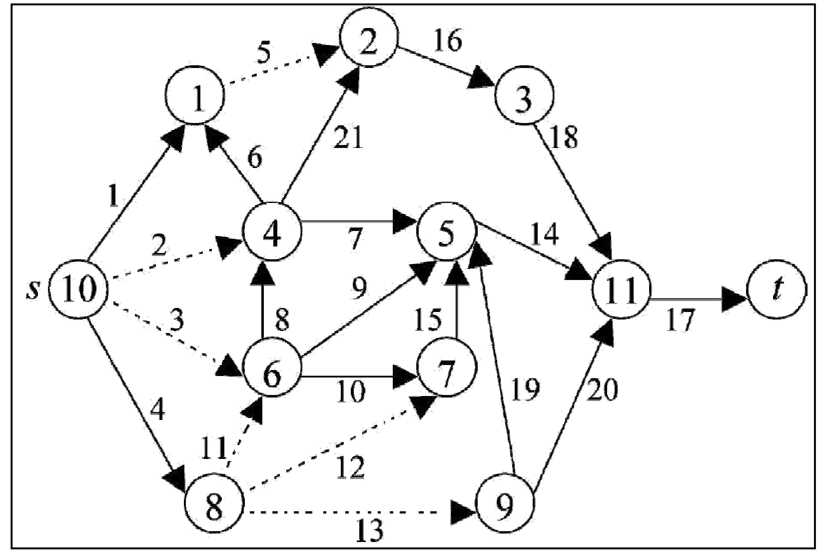
(Ramirez-Marquez and Coit, 2005)

- all 111 MCS's found (10^7 trials, 192 s)

- Most important arcs: 2, 3, 4, 14, 16
  - position with respect to target and source
  - number of downstream arcs (e.g. arcs 1 and 4)
  - failure probability

Influence of failure probability on MCS criticality





MCS 20 $\{14, 16, 20\}$

$frequency = 3 \cdot 10^{-4}$

- More reliable components near target

MCS 7 $\{2, 3, 5, 11, 12, 13\}$

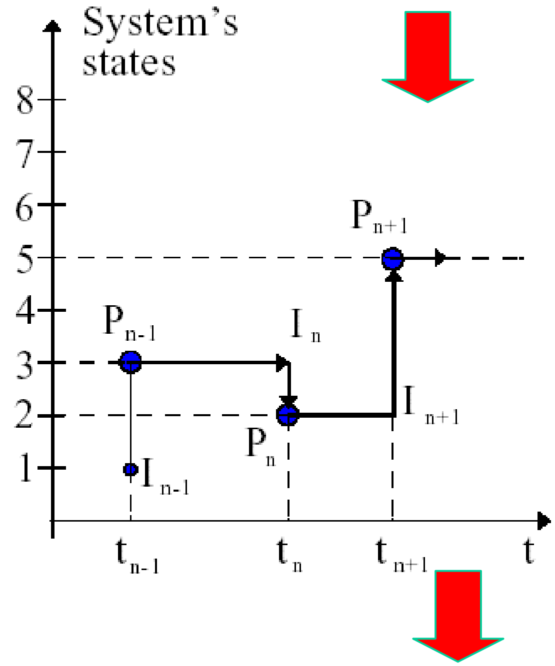
$frequency = 9 \cdot 10^{-4}$

- Less reliable components because of redundancies

BASICS OF MC simulation

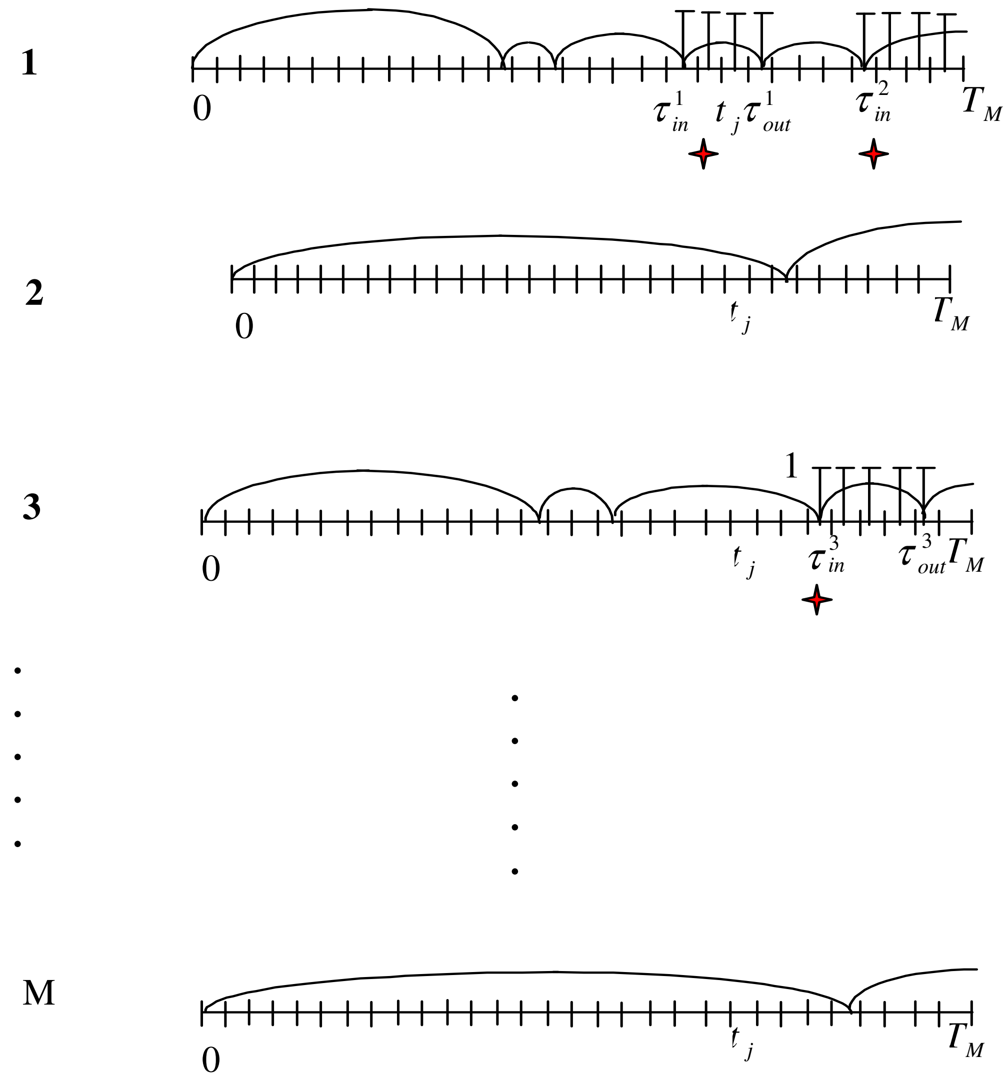
BASICS OF MC Simulation

Procedures for sampling random numbers from given probability distributions



Sampling random walks of arcs failure/repair dynamics

BASICS OF MC Simulation



$$c^A(t_j) = c^A(t_j) + 1$$

$$c^A(t_j) = c^A(t_j)$$

$$c^A(t_j) = c^A(t_j)$$

$$c^A(t_j) = c^A(t_j)$$

$$G^A(t_j) = \frac{c^A(t_j)}{M}$$

Application to network availability

(Zio, Podofillini and Zille, 2005)

Computation of network availability: MC simulation + CA

- Simulate arcs and nodes failure/repair dynamics (MC).

  → sample new configuration of the system after each transition of its elements (nodes and arcs).

- Check system state after each transition (success or failure).

  → check the connectedness between the source and the target nodes (CA).

⇩

Combine Cellular Automata and Monte Carlo Simulation.

Computation of network availability: Method

- The mission time is divided in time channel.
- Simulation of M histories of system life evolution.
- For each history :

  $\Rightarrow$ Sample failure/repair transition times of each network element.

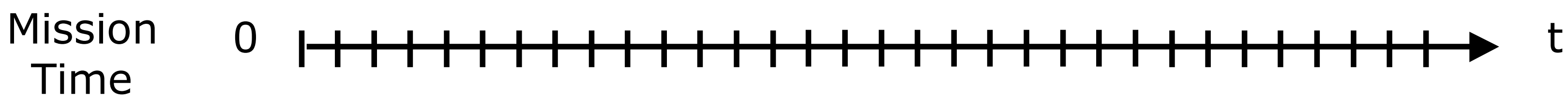
  ⇩

  System configuration in each time channel.

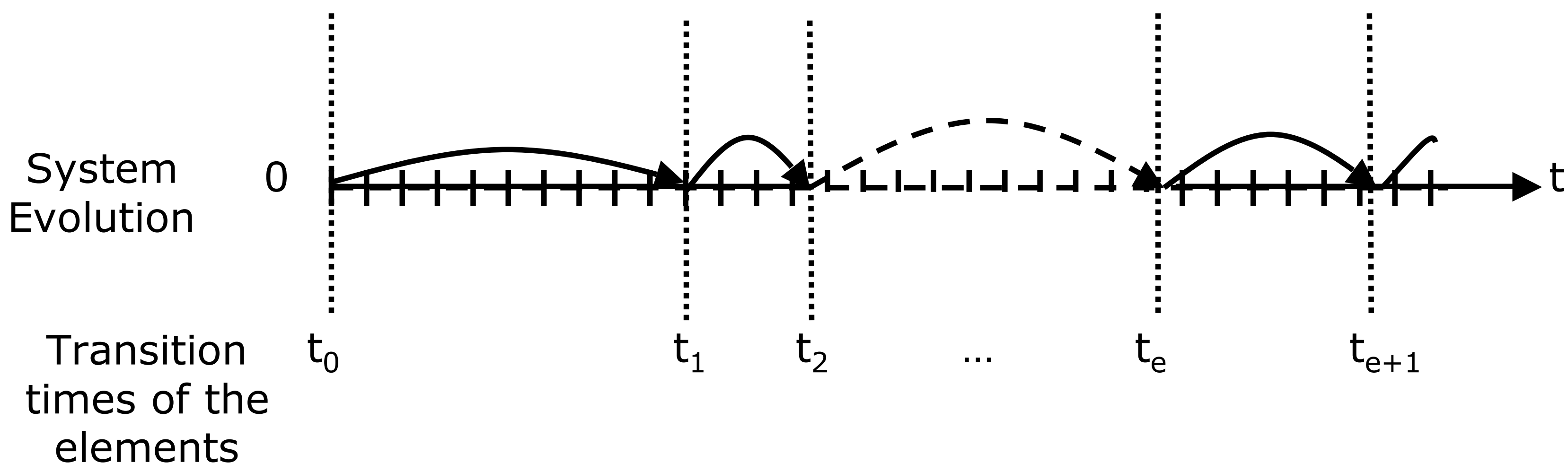
  ⇩

S-T connectedness by CA < successful / failed

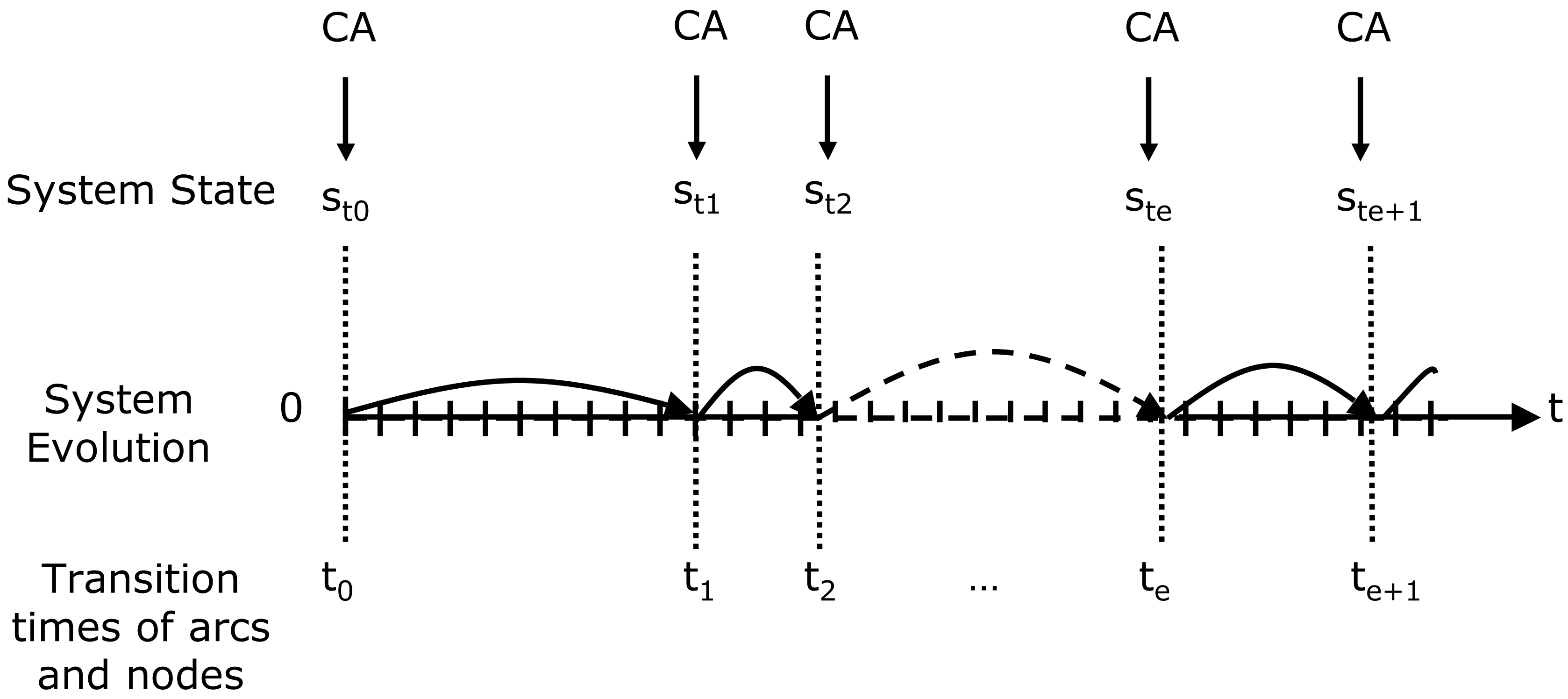
Computation of network availability: Method



Computation of network availability: Method



Computation of network availability: Method



Computation of network availability: Method

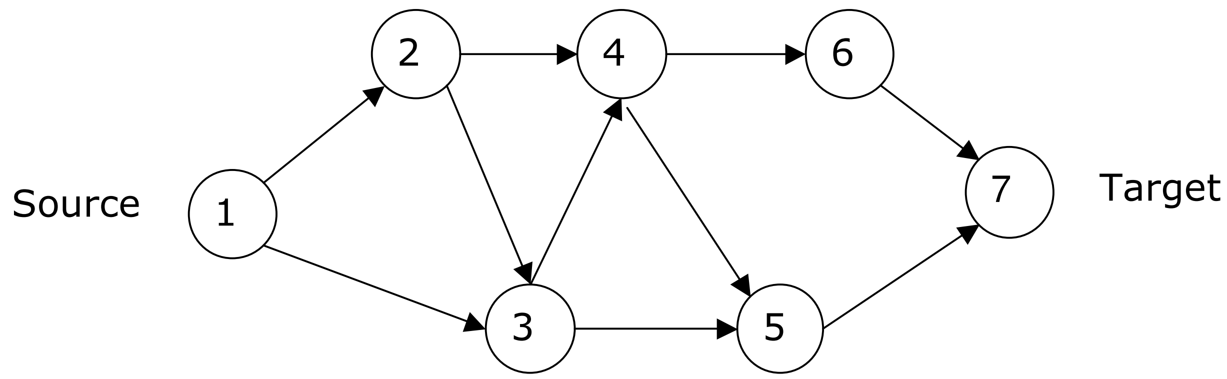
- CA defines the system state after each transition : available or not.

- Collect the portion of time the system is available in the availability counters of the corresponding time channels.

- At the end of the simulation, instantaneous availability of the system :

$$A(t) = \frac{\text{Content of the counter of the corresponding time channel}}{M * dt}$$

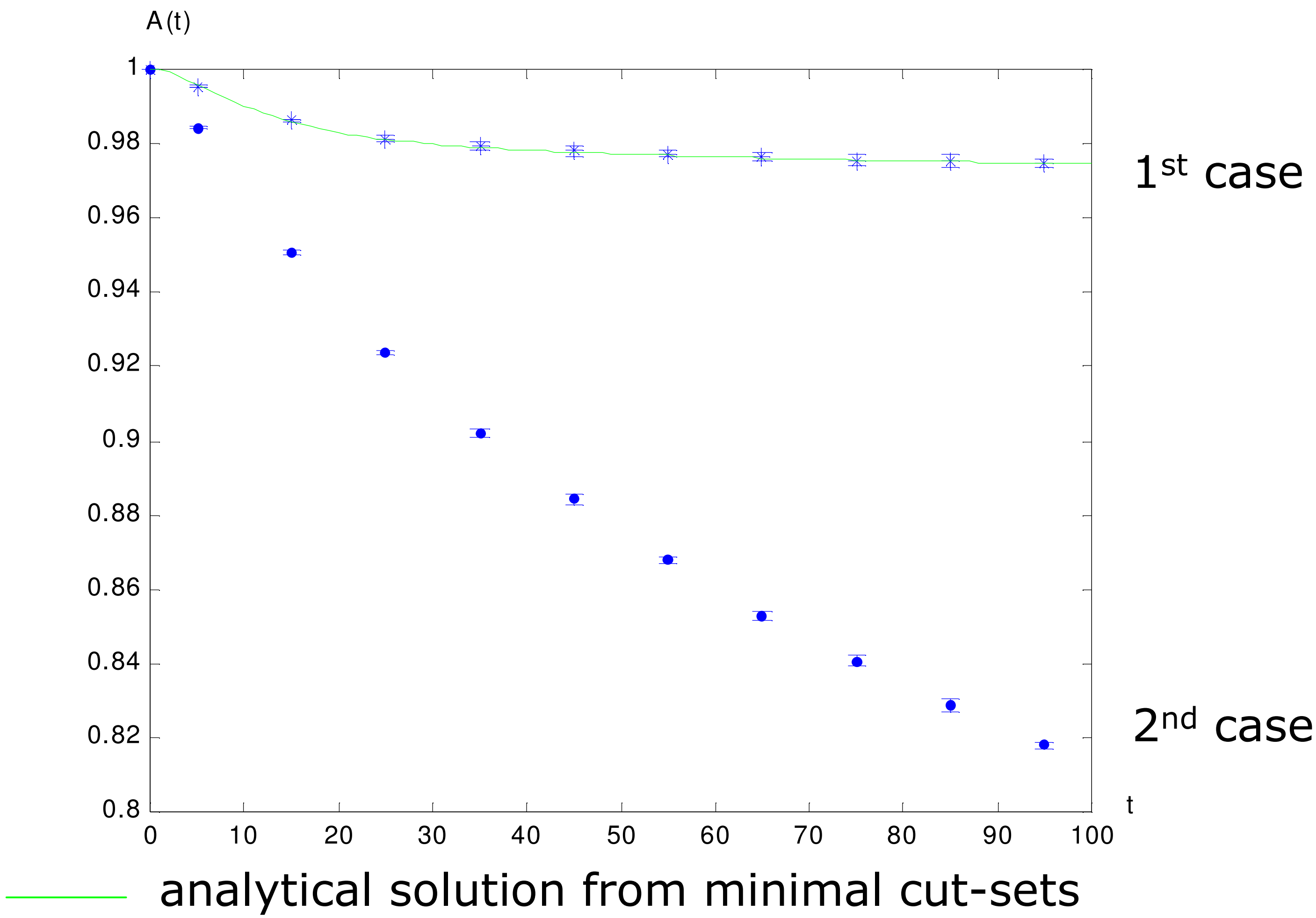
Computation of network availability: Litterature Case Study



1st case : only arcs can fail and be repaired, nodes are assumed perfect.

2nd case : both nodes and arcs can fail and be repaired.

Computation of network availability: Results



CONCLUSIONS

CA+MC:

- ✓ Network connectivity
- ✓ Network reliability
- ✓ Network MCSs e IMs
- ✓ Network Availability

OTHER DEVELOPMENTS

(Zio, Librizzi and Sansavini, 2006)

<u>Binary networks</u> ⟹ <u>Multi-state networks</u>

arc state → 0, 1

arc state → 100% (m_i), 90% (m_i-1), ⋮, 0% (1)

Two-terminal reliability (<u>2TR</u>) ⟹ Multi-state two-terminal reliability at demand level d (<u>M2TR$_d$</u>)

Minimal cut sets (<u>MCS</u>) ⟹ Multi-state minimal cut vectors (<u>MMCV</u>)

Highly reliable networks - Biased Monte Carlo

- High reliability $\Rightarrow$ few failure occurrences $\Rightarrow$ bad statistics
- Biasing $\Rightarrow$ failures favoured $\Rightarrow$ variance reduction

<u>Basic algorithm</u>

i: arc, *j*: state

1. Increase probabilities of arcs in low performing states, $p_{i,j} \rightarrow p^{*}_{i,j}$
2. Sample network configuration from $p^{*}_{i,j}$
3. Compare with network MMCVs (assumed given)
4. If failed configuration $\Rightarrow$ accumulate <u>weight</u>
5. Compute sample estimates of unreliability and variance

$$\prod_{i=1}^{n_{arcs}} \frac{p_{i,j}}{p^{*}_{i,j}}$$

Biased Monte Carlo simulation

Specific biasing

w = mean arc performance in the system

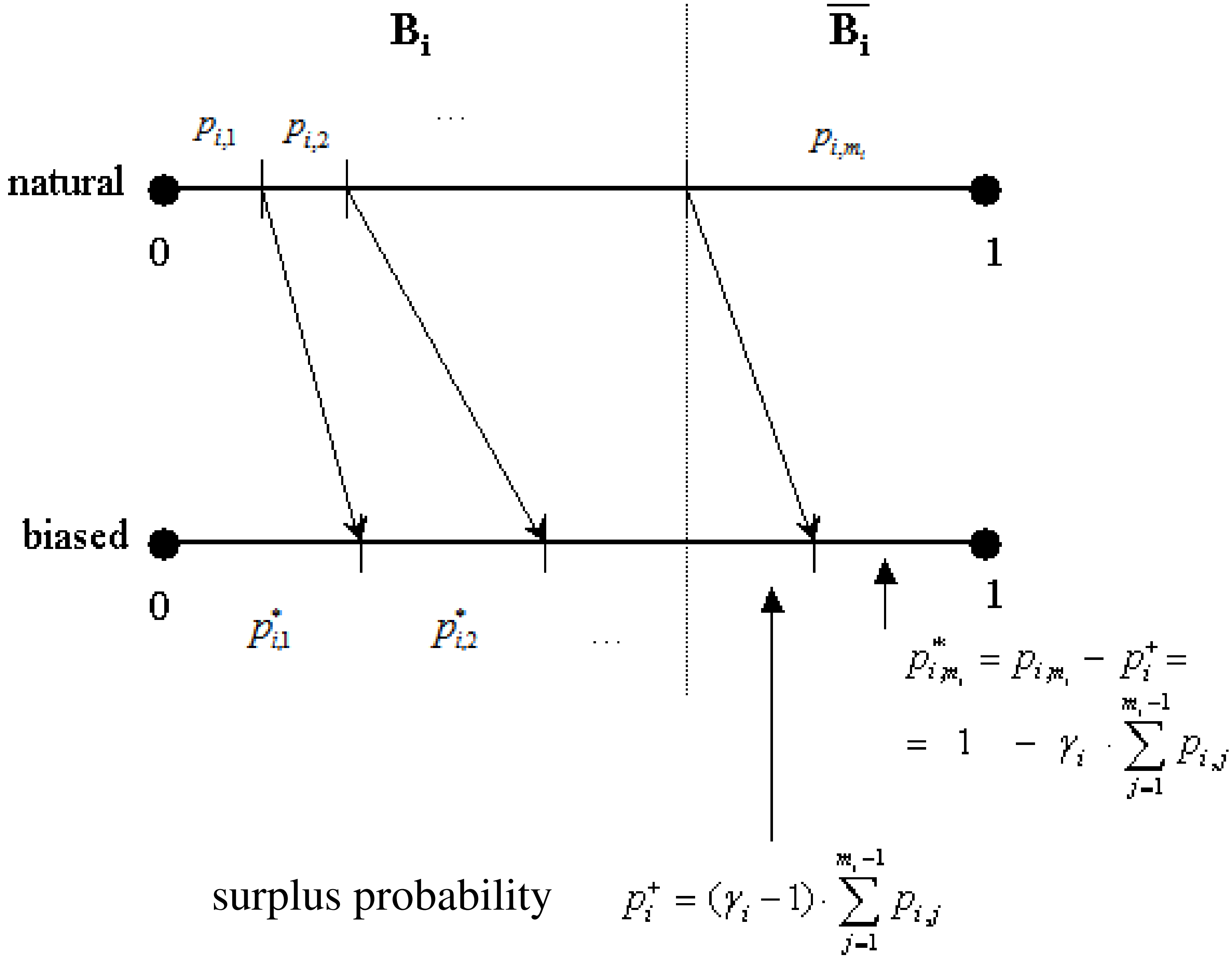
- Set threshold for biasing: $w_{th} = k \cdot w$

- Bias arc i with $w_i > w_{th}$ so that $w_i \le w_{th}$

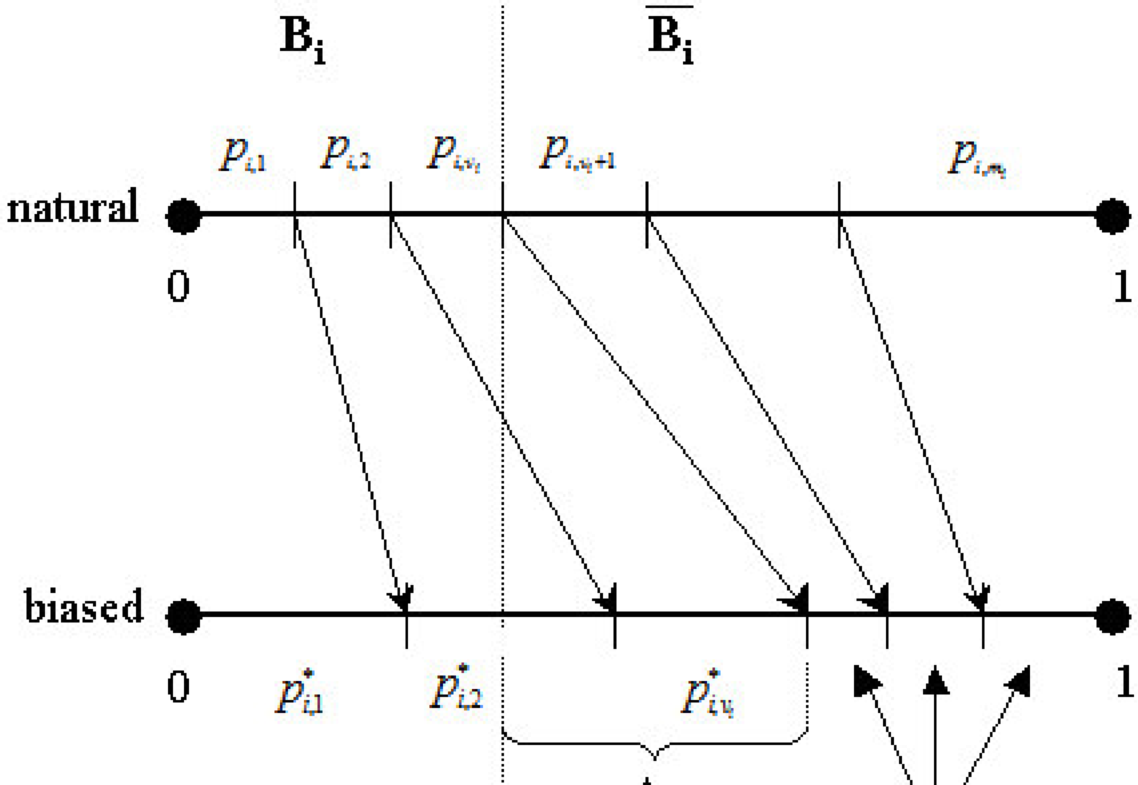
Method 1

Method 2

Biasing method 1 – all states below the nominal one



Biasing method 2 – all states below the minimal in MMCV



$$p_i^+ = (\gamma_i - 1)\cdot \sum_{j\in B_i} p_{i,j}$$

$$p^*_{i,j} = p_{i,j} - \frac{p_{i,j}}{\sum_{j'\in \overline{B_i}} p_{i,j'}}\cdot p_i^+ = p_{i,j}\cdot\left[\frac{1-\gamma_i\cdot\sum_{j'\in B_i} p_{i,j'}}{\sum_{j'\in \overline{B_i}} p_{i,j'}}\right]$$

only failed states are forced

Conclusions

- Need for efficient computational techniques for assessing highly reliable, multi-state networks $\Rightarrow$ biased MC

Current developments

• Extension CA+MC for application to Security

Zio & Rocco, CNIP' 06, 2006

• Development of new reliability indicators from the evaluation of the network topological properties (Complexity Science)

Zio, 2006

Future developments

MC biasing based on MMCVs, which are difficult to identify

⇓

Flow algorithms, CA (?)

Cellular Automata (CA)

- **WHAT?**
  - ✓ Mathematical models of complex dynamical systems
  - ✓ Large number of identical processing elements with local interactions
  - ✓ Parallel computation
- **WHY?**
  - ✓ Development of computers and computation (von Neumann, 1948)
  - ✓ Models of the dynamics of many real complex systems: e.g. fluids, molecular systems, economical systems, ecological systems

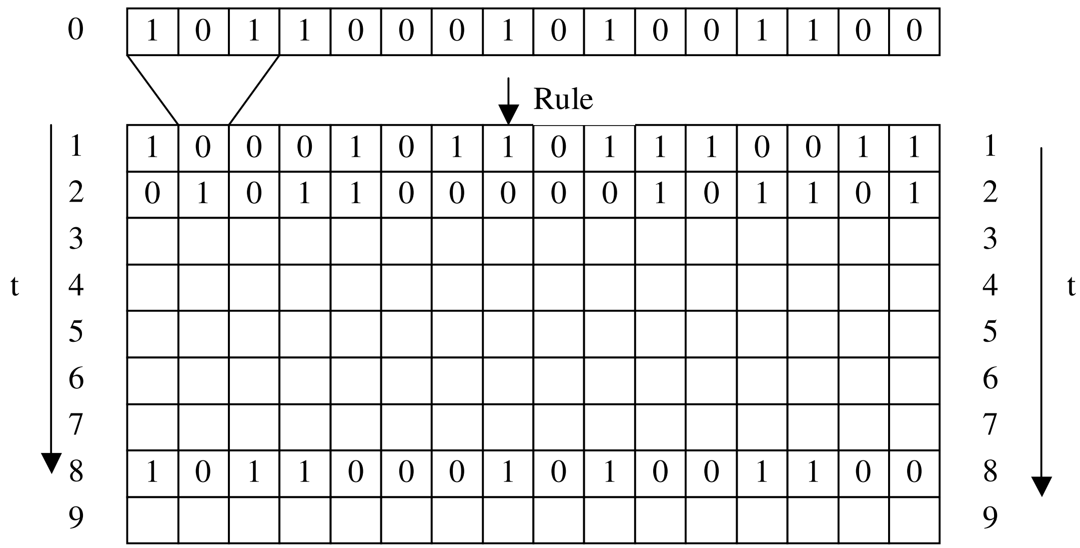
BASICS OF CA

BASICS OF CA

1. Spatially- and temporally-discrete
2. Local interaction
3. Parallel evolution

BASICS OF CA
1. Spatially- and temporally-discrete

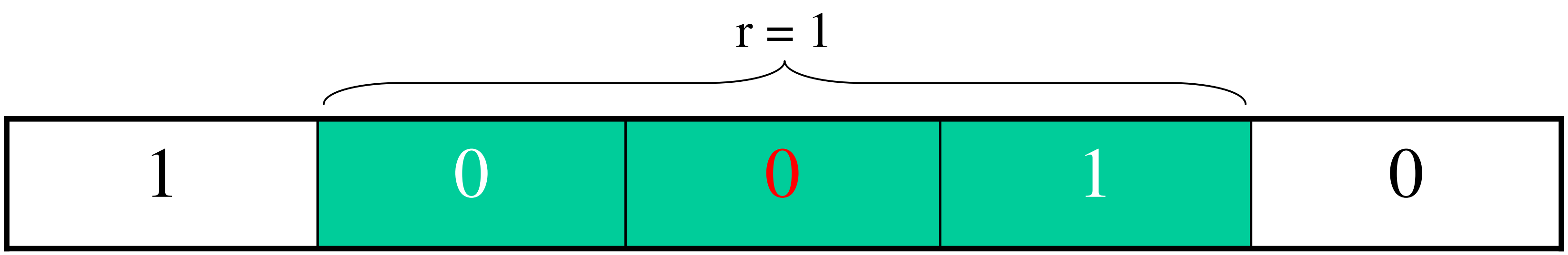
- ✓ $\mathcal{L}$ = discrete lattice of cells (state space for CA dynamics)



- ✓ Each cell of $\mathcal{L}$ is a *finite automaton* which assumes values in S≡{0,1,2,…,k-1}
- ✓ $s_i(t)$= state of the cell *i (1D)* at the discrete time *t*
- ✓ $\mathcal{L}$ is homogeneous: all cells bear the same properties

BASICS OF CA
2. Local interaction

- ✓ N_i = predefined local neighbourhood of cell i



- ✓ Cell i interacts only with the n cells in N_i
- ✓ Transition rule

$$\phi : \overbrace{S \, X \, S \, X \ldots X \, S}^{n} \rightarrow S$$

$$s_i(t+1) = \phi[s_r(t), \quad r \in N_i]$$

BASICS OF CA
3. Parallel evolution

- ✓ One evolution step of the CA is achieved after the simultaneous application of the rule ϕ to each cell in $\mathcal{L}$

1D-CA Example: Addition modulo 2

- ✓ S≡{0,1}; k=2
- ✓ r=1
- ✓ 1st order in time

$$s_i(t+1) = \mathrm{mod}_2\left[s_{i-1}(t) + s_i(t) + s_{i+1}(t)\right] \equiv \oplus_2\left\{s_{i-1}(t), s_i(t), s_{i+1}(t)\right\}$$

$i-1(t), i(t), i+1(t)$	1,1,1	1,1,0	1,0,1	1,0,0	0,1,1	0,1,0	0,0,1	0,0,0
$i(t+1)$	1	0	0	1	0	1	1	0

1D-CA Example: Addition modulo 2

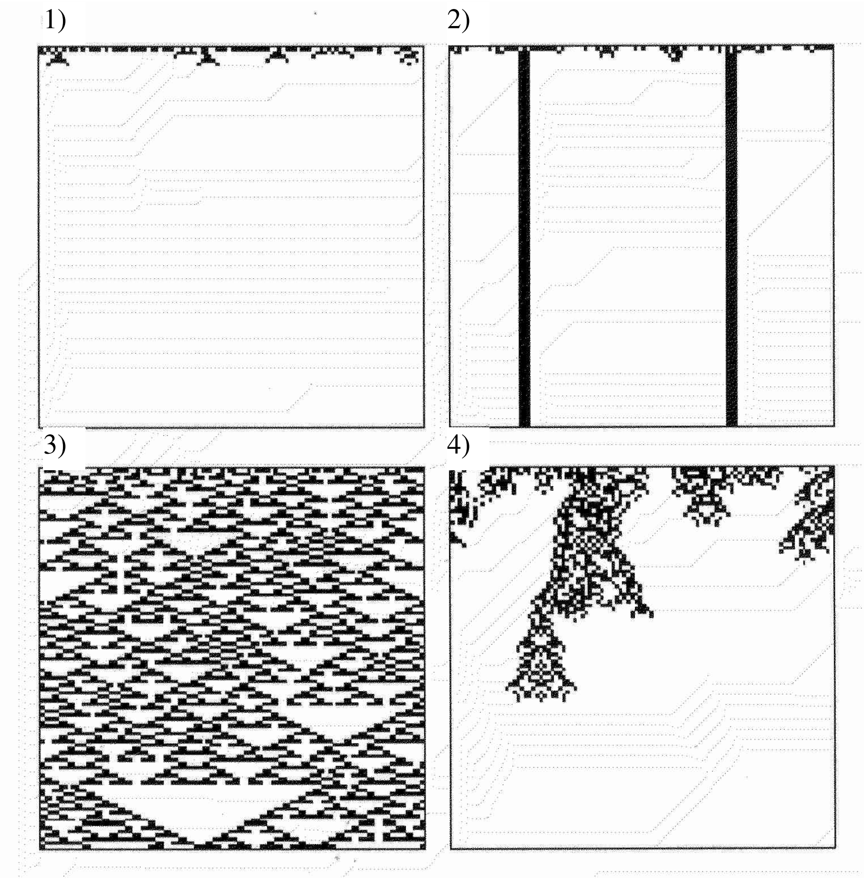
t																
0	1	0	1	1	0	0	0	1	0	1	0	0	1	1	0	0
1	1	0	0	0	1	0	1	1	0	1	1	1	0	0	1	1
2	0	1	0	1	1	0	0	0	0	0	1	0	1	1	0	1
3																
4																
5																
6																
7																
8	1	0	1	1	0	0	0	1	0	1	0	0	1	1	0	0
9																

Rule

t

CA Behavioural classes

1. fixed points
2. inhomogeneous configuration or cycles
3. chaotic, aperiodic patterns
4. complex, localized, propagating structures



CA vs DE

- ✓CA can be considered an alternative to DE
- ✓Discretization for numerical solution of DE ~ local discrete dynamical system of CA
- ✓DE can lead to analytic solutions in simple cases
- ✓CA are more convenient for simulation
- ✓HOWEVER: setting up a CA corresponding to a DE is a difficult problem ⇨phenomenology

BASICS OF MC sampling

BASICS OF MC Sampling

Procedures for sampling random numbers from given probability distributions

↓

"Coin Toss":

p_{ji} = probability of arc *ji* "success"

q_{ji} = probability of arc *ji* "failure"

↓

Sampling realizations of arcs failure/success configurations